17. *Singularities of the Solution of a Non-linear Wave Equation*

By Sadakazu AIZAWA

Department of Mathematics, Kōbe University

(Comm. by K. KUNUGI, M.J.A., March 12, 1962)

1. Concerning the characteristic initial value problem for the non-linear wave equation

$$\square u \equiv \frac{\partial^2 u}{\partial x_1^2} - \frac{\partial^2 u}{\partial x_2^2} - \frac{\partial^2 u}{\partial x_3^2} = f(x_1, x_2, x_3, u) \tag{1}$$

in two space variables, the author [1] proved the existence of a generalized solution satisfying vanishing initial condition under the assumption that i) $f(x, u)$ is continuous in $\mathfrak{D}$, and ii) the inequality $\square \underline{\omega}(x) \leq f(x, u) \leq \square \overline{\omega}(x)$ holds in $\mathfrak{D}$, where

$$\mathfrak{D} = \{(x, u);\, x \in \overline{D},\, \underline{\omega}(x) \leq u \leq \overline{\omega}(x)\}.^{1)}$$

In this note we are concerned with singularities of the solution of the characteristic initial value problem for (1), that is, we shall show in §3 that for a certain class of functions $f(x, u)$, the solution of (1) with vanishing initial condition becomes infinite at a (finite) point in D. The proof is similar to that given by J. B. Keller [2] for the solution of the Dirichlet problem concerning the non-linear elliptic equation $\Delta u = f(u)$.

2. To give an explicit bound on the solution of (1), we shall make use of the following comparison theorem.

THEOREM 1. *Let $f(x, u)$ and $\underline{f}(x, u)$ be continuous functions defined for $x \in \overline{D}$ and $-\infty < u < +\infty$, and let the inequality $f(x, u) \geq \underline{f}(x, \underline{u})$ hold for $x \in \overline{D}$ and $u \geq \underline{u}$. Further let $\underline{f}(x, u)$ be Lipschitz continuous*[2] *with respect to u.*

Assume that $u(x)$ and $\underline{u}(x)$ are generalized solutions in D of the equations $\square u = f(x, u)$, $\square \underline{u} = \underline{f}(x, \underline{u})$ with initial conditions $u(x) = \varphi(x)$, $\underline{u}(x) = \underline{\varphi}(x)$ on S_Σ respectively, where $\varphi(0) \geq \underline{\varphi}(0)$ and $\partial\varphi/\partial\lambda_x \geq \partial\underline{\varphi}/\partial\lambda_x$ on S_Σ. Then the inequality $u(x) \geq \underline{u}(x)$ holds in $\overline{D}$.

For the proof, see Theorem 2.3 in [1].

3. We begin by considering the ordinary non-linear differential equation of the second order

$$\frac{d^2 v}{dr^2} + \frac{\lambda}{r}\frac{dv}{dr} = h(r, v) \quad (\lambda > 0) \tag{2}$$

1) For the notation refer to S. Aizawa: Differentiability of the generalized solution of a non-linear wave equation, Proc. Japan Acad., **38**, 69-74 (1962).

2) $|\underline{f}(x, u_1) - \underline{f}(x, u_2)| \leq L(M)\,|u_1 - u_2|$ provided $|u_1|$, $|u_2| \leq M$.