35. *Decomposition of Kronecker Products of Representations of the Inhomogeneous Lorentz Group*

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(Comm. by K. KUNUGI, M.J.A., April 12, 1962)

We shall consider irreducible decompositon of the Kronecker products of unitary representations of the proper orthochronous inhomogeneous Lorentz group which we shall denote by G. In the present paper we give explicit solution of this problem, except some cases. Our method is based on the theory of induced representation, which was considered by G. W. Mackey [1]. The details of the results will be reported in another paper.

§1. It is known that all irreducible unitary representations of G are classified in the following types.

a) $\mathfrak{D}^0(\sigma)$. Let H be the homogeneous Lorentz group. We can consider it the factor group of G by the real four-dimensional vector subgroup N of G. We can define canonically an irreducible representation $\mathfrak{D}^0(\sigma)$ of G from an irreducible representation σ of the factor group H. σ is characterized by a pair of parameters (s, t), according whose values σ's are classified into three series. (i) Principal series: s is a non-negative integer, t is pure imaginary. (ii) Supplementary series: $s=0$, $0<t<1$. (iii) Identity representation: $(s,t)=(0,1)$.

b) $\mathfrak{D}^1(\rho, b)$. Consider the three-dimensional rotation group R, which is a subgroup of H. Its irreducible representation $\rho\equiv\rho(m)$ is decided by the highest weight m, which is a non-negative integer. We choose a character $\chi(x)=\exp(ibx_1)$ $(b\neq 0)$ for the element $x=(x_1, x_2, x_3, x_0)$ of N, and construct a representation of RN as the product of $\rho(m)$ and $\chi(x)$, and lastly we induce a unitary representation $\mathfrak{D}^1(\rho, b)$ of G from this representation of RN. Then $\mathfrak{D}^1(\rho, b)$ is irreducible.

c) $\mathfrak{D}^2(\lambda, c)$. In the case b), we replace R with the three-dimensional proper Lorentz group L, the representation ρ of R with an irreducible representation λ of L and the character $\exp(ibx_1)$ of N with $\exp(icx_0)$ $(c>0)$ respectively. Then the induced representation $\mathfrak{D}^2(\lambda, c)$ from $\exp(icx_0)\lambda$ of LN is irreducible. There are four series of irreducible representations λ of L: (i) Principal series: λ^l $(1/4\leq l)$. (ii) Supplementary series: λ^l $(0<l<1/4)$. (iii) Discrete series: λ_p $(p=\pm 1, \pm 2,\cdots)$. (iv) Identity representation: I.

d) $\mathfrak{D}^3(\kappa, +)$ and $\mathfrak{D}^3(\kappa, -)$. We employ the motion group M over two-dimensional Euclidean space as the third subgroup of G, and let κ