33. *Existence Theorems on Difference-Differential Equations*

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As an application of a fixed point theorem due to Tychonov, the author [2] has obtained a theorem concerning the existence of solutions of difference-differential equations defined on a finite interval of t such that

$$x'(t)=f(t, x(t), x(t-1))$$

under the initial conditions $x(t-1)=\varphi(t)$ $(0\leqq t<1)$ and $x(0)=x_0$, where $\varphi(t)$ is a given continuous function. In [2], he imposed on $f(t,x,y)$ only the condition of continuity of $f(t,x,y)$ in (t,x,y). For the practical problems defined on an infinite interval of t, the function $f(t,x,y)$ has so restricted a form that in the sequel we shall consider the equations, in which the function f has some stronger restrictions than those in [2].

The purpose of this paper is to obtain some results concerning the existence, stability, and boundedness of solutions of difference-differential equations by making use of Tychonov's fixed point theorem.

Recently, as an application of Tychonov's theorem, Stokes [1] has discussed the same problems as above for nonlinear differential equations. His method can also be applied for difference-differential equations.

We first prove the following

THEOREM 1. *Let $F(t,x,y)$ be continuous and nonnegative in (t,x,y) and nondecreasing in x and y for fixed t in the region R defined by $0\leqq t<\infty$ and $0\leqq x\leqq f(t)$, $0\leqq y\leqq f(t)$*[1] *where*

(i) *$f(t)$ is continuous in the interval I: $0\leqq t<\infty$ and $f(0)=\alpha$ $(\geqq 0)$;*

(ii) *$f(t)$ satisfies a difference-differential inequality*

$$f'(t)\geqq F(t, f(t), f(t-1))$$

under the condition $f(t-1)=|\varphi(t)|$ $(0\leqq t<1)$ for a given continuous function $\varphi(t)$, which has the limit $\lim_{t\to 1-0}\varphi(t)$.

Then, if we define a transformation T such that

$$Tf(t)=\alpha+\int_0^t F(s, f(s), f(s-1))\,ds,$$

1) As usual, $F(t,x,y)$ may be continuously extended to the whole region $|x|<\infty$, $|y|<\infty$. (Cf. [2].)