40. 2-Primary Components of the Homotopy Groups of Spheres

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(Comm. by Z. SUETUNA, M.J.A., May 12, 1962)

This is a preliminary report of results concerning the generators of 2-primary components of the homotopy groups of spheres. The proofs will be given elsewhere.

1. Let E: $\pi_m(S^n)\to\pi_{m+1}(S^{n+1})$ be the suspension homomorphism. If α_n is an essential element of $\pi_m(S^n)$, we shall denote $E\alpha_n$ by α_{n+1}. The homotopy class of the identity map $S^n\to S^n$ is denoted by ι_n. Let $\alpha\in\pi_p(S^n)$, $\beta\in\pi_q(S^n)$. Suppose that there exists a map $h: S^p\times S^q\to S^n$ of type (α,β), then we denote by (α,β) the coset of the subgroup $E\pi_{p+q}(S^n)$ of $\pi_{p+q+1}(S^{n+1})$ which includes the homotopy class of the map obtained by Hopf construction from h. The triad Whitehead product of α and β will be denoted by $\{\alpha,\beta\}\in\pi_{p+q+1}(S^{n+1}; E_+, E_-)$ ([4] or [5]). Define a homomorphism $P: \pi_m(S^n)\to\pi_{m+n+1}(S^{n+1}; E_+, E_-)$ by $P(\alpha)=\{\alpha,\iota_n\}$ for $\alpha\in\pi_m(S^n)(n\geqq 2)$.

We use the notation R_n instead of $SO(n)$. G. W. Whitehead defined a homomorphism $J: \pi_m(R_n)\to\pi_{m+n}(S^n)$ ([6]). We can prove

(1.1) $J(\alpha\circ\beta)=J(\alpha)\circ E^n\beta$ for $\beta\in\pi_p(S^m)$, $\alpha\in\pi_m(R_n)$,

and that in the diagram:

(1.2)
$$\begin{array}{ccccccc}
\cdots\pi_m(R_n) & \xrightarrow{j^*} & \pi_m(R_{n+1}) & \xrightarrow{p^*} & \pi_m(S_n) & \xrightarrow{\tilde{\Delta}} & \pi_{m-1}(R_n) \\
J\downarrow & & \downarrow J & & \downarrow P & & \downarrow J \\
\cdots\to\pi_{m+n}(S^n) & \xrightarrow{E} & \pi_{m+n+1}(S^{n+1}) & \xrightarrow{i^*} & \pi_{m+n+1}(S^{n+1}; E_+, E_-) & \xrightarrow{\Delta} & \pi_{m+n-1}(S_n)
\end{array}$$

the upper sequence is the bundle sequence of $R_{n+1}\to R_{n+1}/R_n$, and the lower sequence is the suspension sequence of S^n. We can also prove that the following relations hold:

(1.3) (a) $E\circ J=J\circ j^*$, (b) $i^*\circ J=P\circ p^*$ (c) $\Delta\circ P=-J\circ\tilde{\Delta}$.

I. M. James [4] defined a homomorphism $H: \pi_m(S^n)\to\pi_m(S^{2n-1})$, which is a generalization of the Hopf-invariant.

P. J. Hilton [7] also defined a homomorphism $\hat{H}: \pi_m(S^n)\to\pi_m(S^{2n-1})$ in a different way. I owe M. G. Barratt the announcement that H is the same as $\hat{H}$.

We denote by $\{\alpha,\beta,\gamma\}$ and $\{\alpha,\beta,\gamma,\delta\}$ Toda's constructions, ([1], [2], [3]).

The homotopy groups $\pi_{n+r}(S^n)$, $r\leqq 7$, are well known. We list