61. On Circle and Quasi-Hausdorff Methods of Summability of Fourier Series

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§1. Euler and circle methods of summability of Fourier series. Here the author wishes to discuss the circle method of summability and other quasi-Hausdorff methods of summability of Fourier series. At the beginning we remember the Euler method of summability. It associates with a given sequence $\{s_n\}$ the means

$$\sigma_{n,r}=\sigma_n=\sum_{\nu=0}^{n}\binom{n}{\nu}r^\nu(1-r)^{n-\nu}s_\nu,\quad n=0,1,2,\cdots,$$

where r is a constant which satisfies $0<r\leq 1$. We denote this method as (ε, r). The case $r=1$ corresponds to ordinary convergence. The Lebesgue constants for this method of Fourier series are given by L. Lorch [1] and A. E. Livingston [2].

Theorem 1. *The Lebesgue constants for the* (ε, r) *method are given by*

$$L(n;r)=\frac{2}{\pi^2}\log\left|\frac{2nr}{1-r}\right|+A+o(1),\ \textit{as}\ n\to\infty,\ \textit{where}$$

$$A=-\frac{2C}{\pi^2}+\frac{2}{\pi}\int_0^1\frac{\sin u}{u}\,du-\frac{2}{\pi}\int_1^\infty\left\{\frac{2}{\pi}-|\sin u|\right\}\frac{du}{u}.$$

C is the Euler-Mascheroni constant.

The Gibbs phenomenon of the Fourier series $\sum_{n=1}^{\infty}\frac{\sin nt}{n}$ for this method are investigated by O. Szász [3].

Theorem 2. *If we put* $s_0=0$, $s_n=\sum_{\nu=1}^{n}\frac{\sin \nu t}{\nu}$, *then we have*

$$\lim_{n\to\infty}\sigma_n(t_n)=\int_0^{\tau r}\frac{\sin y}{y}\,dy,\ \textit{as}\ nt_n\to\tau\ \textit{and}\ nt_n^2\to 0.$$

On the other hand the circle method of summability associates with a given sequence $\{s_n\}$ the means

$$\sigma^*_{n,r}=\sigma^*_n=\sum_{\nu=n}^{\infty}\binom{\nu}{n}r^{n+1}(1-r)^{\nu-n}s_\nu,\quad n=0,1,2,\cdots,$$

where r is a constant which satisfies $0<r\leq 1$. The case $r=1$ corresponds to ordinary convergence. We denote this method as (γ, r). The Lebesgue constants for this method of Fourier series are given by the author [4].

Theorem 3. *The Lebesgue constants for the* (γ, r) *method are given by*