57. On Irreducible Representations of the Lorentz Group of n-th Order

By Takeshi HIRAI
Department of Mathematics, University of Kyoto
(Comm. by K. KUNUGI, M.J.A., June 12, 1962)

Let L_n be the Lorentz group of n-th order, i.e. the connected component of the identity element of the group of all homogeneous linear transformations in the real n-dimensional vector space which leave the quadratic form $x_1^2+x_2^2+\cdots+x_{n-1}^2-x_n^2$ invariant.

The formulas for infinitesimal operators of the irreducible representations of L_n were indicated in the paper [1]. In the present paper we classify irreducible representations of L_n and distinguish unitary ones by the results obtained in [1]. We consider also two-valued representations. Moreover it is not difficult to distinguish irreducible representations which leave Hermitian forms invariant and to investigate these Hermitian forms.

The author expresses his hearty thanks to Professor H. Yoshizawa who has encouraged him with kind discussions.

§1. **Preliminaries.** We use same definitions and notations as in [1]. We consider the irreducible representations $\{T_g, H\}$ which are differentiable and satisfy the assumption (U). These are determined by their $(n-1)$-infinitesimal operators $A_{2,1}, A_{3,2},\cdots, A_{n-1,n-2}$ and $B=B_{n-1}$ corresponding to the one-parameter subgroups $g_{2,1}(t)$, $g_{3,2}(t),\cdots, g_{n-1,n-2}(t)$ and $g_{n-1}(t)$ respectively. The subgroups $g_{i,i-1}(t)$ $(2\leq i\leq n-1)$ generate a maximal compact subgroup U_n (rotation group in the space $x_n=0$) and the operators $A_{i,i-1}(2\leq i\leq n-1)$ determine the representation of U_n which is induced from $\{T_g, H\}$. This representation of U_n can be decomposed into irreducible components. The operator B is determined by a row of $[n/2]-1$ integers $\alpha=(n_1, n_2,\cdots, n_{[n/2]-1})$ and a complex number c.

It is easy to see that an irreducible representation of L_n is characterized by parameters $(\alpha; c)$ in the operator B and a set of irreducible representations β of U_n which is contained in the induced representation. To every generic value $(\alpha; c)$ of parameters there corresponds one irreducible representation of L_n, and in exceptional cases two or three ones. It may be of some interest to discuss this correspondence. In these arguments it is sufficient to consider only one operator B.

§2. **Classification of irreducible representations.** There are remarkable differences according to the parity of n.