56. *On Quasi-Denjoy Integration*

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1. Introduction and terminology. We are concerned with showing that the Denjoy-Khintchine process of integration for functions of one real variable is capable of an essential generalization. The gist of our theory consists in replacing the class of the generalized absolutely continuous functions by a broader one composed of the functions called generalized highly continuous in our phrasing.

As regards general terminology (and notation), we shall conform on the whole to the *Theory of the Integral* by Saks, except in certain minor points. The mentioned treatise will be quoted hereafter simply as Saks for short. The conventions that follow will be valid throughout. By *sets* and *intervals*, by themselves, we shall always understand linear sets and linear non-degenerate intervals respectively, where intervals may be infinite (i.e. unbounded). The epithets *open* and *closed* for intervals will as usual be applied only to finite intervals. The term *function* will stand exclusively for a point-function defined on the whole real line and assuming finite real values, unless another meaning is obvious from the context. Finally, a *sequence* will mean a nonvoid countable one, finite or infinite.

2. Semiabsolutely and strongly semiabsolutely continuous functions. Let $F(x)$ be a function, E a set, and α a number such that $0<\alpha\leqq 1$. We say that F is *semiabsolutely continuous* (α) on E, or briefly SC(α) on E, iff (i.e. if and only if) given any $\varepsilon>0$ there is an $\eta>0$ such that for every finite sequence of non-overlapping closed intervals $I_1,\cdots,I_n$ whose extremities belong to E, the inequality

$$|I_1|^\alpha+\cdots+|I_n|^\alpha<\eta \quad \text{implies} \quad |F(I_1)|+\cdots+|F(I_n)|<\varepsilon.$$

Remember that whenever I is a closed interval, $F(I)$ denotes the increment of the function $F(x)$ over I, while the image of I under F will be written $F[I]$ (see Saks, p. 99 and p. 100). When especially $\alpha=1$, the notion just introduced plainly reduces to the absolute continuity on E of the function F (Saks, p. 223).

We say further that F is *strongly semiabsolutely continuous* (α), or SSC(α), on E, iff F is SC(α) on E and moreover the set E is of α-dimensional volume zero (Saks, p. 54). When this is the case, E is evidently a set of measure zero (in the Lebesgue sense).

The reference to the exponent α will be omitted in the above two notions when we are interested only in the existence of α, not