54. A Remark on Convexity Theorems for Fourier Series

By Kenji YANO
Department of Mathematics, Nara Women's University, Nara
(Comm. by Z. SUETUNA, M.J.A., June 12, 1962)

In the previous paper [1], we have proved a number of convexity theorems concerning Fourier series. In the present paper, we shall improve some of them replacing either of the conditions by one-sided one.

Let $\varphi(t)$ be an even function integrable in $(0,\pi)$ in Lebesgue sense, periodic of period 2π, and let

$$\varphi(t)\sim\frac{1}{2}a_0+\sum_{n=1}^{\infty}a_n\cos nt,$$

and

$$\Phi_0(t)=\varphi(t),\ \Phi_\alpha(t)=\frac{1}{\Gamma(\alpha)}\int_0^t(t-u)^{\alpha-1}\varphi(u)du\ (\alpha>0).$$

The (C,β) sum of the Fourier series of $\varphi(t)$ at $t=0$ is

$$s_n^\beta=A_n^\beta\frac{1}{2}a_0+\sum_{\nu=1}^{n}A_{n-\nu}^\beta a_\nu=\sum_{\nu=0}^{n}A_{n-\nu}^{\beta-1}s_\nu\ (-\infty<\beta<\infty),$$

where $s_n=s_n^0$, $A_0^\beta=1$ and

$$A_n^\beta=\frac{(\beta+1)(\beta+2)\cdots(\beta+n)}{n!}\ (n\geqq 1).$$

In what follows we understand that $t\to 0$ means $t>0$ and $t\to 0$.

Now, Theorems 2, 4, 5, and 6 in the paper [1] can be improved as follows.

THEOREM 2′. *Let* $0\leqq b, 0<\beta-b\leqq\gamma-c$ *and* $|c-b|<1$. *If as* $t\to 0$,

$$(1)\qquad \int_0^t|\Phi_\beta(u)|du=o(t^{\gamma+1})$$

and

$$\int_0^t(|\Phi_b(u)|-\Phi_b(u))du=O(t^{c+1}),$$

then we have

$$s_n^r=o(n^q),\ q=b+(r-c)\frac{\beta-b}{\gamma-c},$$

as $n\to\infty$, *for* $c<r<\gamma'$, *where*

$$\gamma'=\inf\left(\gamma,\ \frac{(b+1)\gamma-(\beta+1)c}{\gamma-c+b-\beta}\right).$$

COROLLARY 2.1′. *Let* $0<\beta<\gamma$ *and* $0<\delta<1$. *If* (1) *holds, and* $\varphi(t)=O_L(t^{-\delta})$, *then*

$$s_n^\alpha=o(n^\alpha),\ \alpha=\beta\delta/(\gamma-\beta+\delta).$$

THEOREM 4′. *Let* $-1\leqq\beta, 0\leqq c$ *and* $0<\gamma+1-c\leqq\beta+1-b$,