52. 2-Primary Components of the Homotopy Groups of Some Lie Groups

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This is a preliminary report of results concerning the generators of 2-primary components of the homotopy groups of $SO(n)$, $SU(n)$ and $Sp(n)$. The proofs will be given elsewhere.

1. Let $R_n (n \geqq 2)$ denote the special orthogonal group $SO(n)$, $U_n(n \geqq 1)$ the special unitary group $SU(n)$, and $Sp_n(n \geqq 1)$ the symplectic group $Sp(n)$.

Let $i^{m,n} : R_n \to R_m$, $i'^{m,n} : U_n \to U_m$, $i''^{m,n} : Sp_n \to Sp_m$, $(n \leqq m)$
$l^{2n} : Sp_n \to U_{2n}$, $k^{2n} : U_n \to R_{2n}$, $(n \geqq 1)$,
be the inclusion maps.

Let us denote the projections and the characteristic classes of the bundles R_{n+1}, U_{n+1}, and Sp_{n+1}, by
$p : R_{n+1} \to S^n$, $p' : U_{n+1} \to S^{2n+1}$, $p'' : Sp_{n+1} \to S^{4n+3}$,
$T_n \in \pi_{n-1}(R_n)$, $T'_n \in \pi_{2n}(U_n)$, $T''_n \in \pi_{4n+2}(Sp_n)$,
respectively.

Let Z, Q, and C denote the field of complex numbers, algebras of quaternions, and of Cayley numbers over the field of real numbers, respectively.

Then the spheres S^1, S^2, S^3, S^6, and S^7 are represented as follows:
$S^1 = \{z \in Z;\ z\bar{z} = 1\}$, $S^3 = \{q \in Q;\ q\bar{q} = 1\}$, $S^7 = \{c \in C;\ c\bar{c} = 1\}$,
$S^2\{q' = x_1k + x_2j + x_3i + x_4 \in S^3;\ x_4 = 0\}$,
$S^6 = \{c = (q, q') \in S^7;\ x_8 = 0\}$, where $q' = x_5k + x_6j + x_7i + x_8$.

Define the maps
$\sigma'^1_1 : S^1 \to U_1$, $\sigma^2_1 : S^1 \to R_2$, $\sigma''^1_3 : S^3 \to Sp_1$, $\sigma'^2_3 : S^3 \to U_2$,
$\sigma^4_3 : S^3 \to R_4$, $\sigma^8_7 : S^7 \to R_8$, $\rho^3_3 : S^3 \to R_3$, $\rho^7_7 : S^7 \to B_7$,
as follows:
$\sigma'^1_1(z)(z') = zz'$, $(z, z' \in S^1)$, $\sigma^2_1 = k^2 \circ \sigma'^1_1$,
$\sigma''^1_3(q)(q') = qq'$ $(q, q' \in S^3)$, $\sigma'^2_3 = l^2 \circ \sigma''^1_3$, $\sigma^4_3 = k^4 \circ \sigma'^2_3$,
$\sigma^8_7(c)(c') = cc'$ $(c, c' \in S^7)$,
$\rho^3_3(q)(q') = qq'\bar{q}$, $(q \in S^3,\ q' \in S^2)$,
$\rho^7_7(c)(c') = cc'\bar{c}$, $(c \in S^7,\ c' \in S^6)$.

We denote e.g. $i^{m,n} \circ \sigma^n_p$ by σ^m_p.

We denote by q and q' the well known isomorphisms:
$q : \pi_m(Sp_2) \to \pi_m(R_5)$, $q' : \pi_m(U_4) \to \pi_m(R_6)$, $(m \geqq 2)$.

Most of generators of the homotopy groups of R_n, U_n and Sp_n, will be represented in terms of the elements defined above and of