77. *On Unified Representation of State Vector in Quantum Field Theory*

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1. **Introduction.** In quantum field theory we must consider the Hilbert space having non-countable bases which corresponds to a sequence of non-negative integers $(n_1, n_2, \cdots)$.

Since we can construct one-to-one mapping from the set of the sequences $(n_1, n_2, \cdots)$ onto the points in $[0, 1]$ interval [8], we can identify these bases to $[0, 1]$ interval.

Let γ denote a point in $[0, 1]$ interval and let ψ_γ be the element of the Hilbert space which corresponds to γ. The element of this Hilbert space is usually represented by the formulae $\int C_\gamma \psi_\gamma d\mu(\gamma)$ and $\sum_{i=1}^{\infty} C_i \psi_{\gamma_i}$, in [3], [4] and [6], where C_i, C_γ are constants, and $d\mu(\gamma)$ is a measure on $[0, 1]$.

By single $d\mu(\gamma)$, however, we cannot represent every element of this Hilbert space. That is to say, by a continuous measure $d\mu(\gamma)$, we cannot represent the element of the second form. On the other hand by the second form, we cannot represent the element of the first form.

In this paper we take a Lebesgue measure $dm(\gamma)$ and represent each element of the Hilbert space by the unified single expression $\int (C_\gamma + C'_\gamma \sqrt{\delta_\gamma})\, dm(\gamma)$ using generalized distributions [7].

Our method of representation uses a L^2-space's closure. But our topology is weaker than L^2-topology.

2. **New topology defined in $L^2[0, 1]$.**

Lemma 1. *There is a one-to-one correspondence between the sequence of non-negative integers $(n_1, n_2, \cdots)$ and the point of interval $[0, 1]$.* [8]

Let's consider the corresponding interval $[0, 1]$. Let $L^2[0, 1]$ denote the space of functions which are defined in the interval $[0, 1]$ and belong to L^2.

Let $\rho_{n, x_0}(x)$ denote the function

$$\rho_{n, x_0}(x) = \begin{cases} 0 \quad \text{for } |x - x_0| \geqq \delta/n \\ kn \exp\{-(\delta/n)^2/((\delta/n)^2 - |x - x_0|^2)\}/\delta \quad \text{for } |x - x_0| < \delta/n, \end{cases}$$

where δ is a positive constant and k is a constant which satisfies the following equality: $k \int_{|x|<1} \exp\{-1/(1-x^2)\}\, dx = 1$.