76. On Distribulions and Spaces of Sequences. II

On the Multiplications of Improper Functions in Quantum Field Theory

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1. Introduction. In quantum field theory, the multiplication of distributions such as $\delta\cdot\delta$, $pfx^{-1}\cdot\delta$, $\delta'\cdot\delta$ etc., plays an important role. [5][6].

But the multiplications of these types can not be defined in proper sense of distribution theory. So, it will be desirable attempt to extend the notion of proper distribution, define extended multiplication and calculate these products. In [8] we introduced the spaces of sequences and generalized distributions and studied the relations between proper distributions and generalized distributions.

The multiplication in spaces of sequences is associative and commutative, which is convenient to the calculation of S matrix in quantum field theory in contrast to the other definitions such as in [4][5][6].

The weak point of the multiplication in spaces of generalized distribution is the indeterminateness of the product. But this defect is not disadvantage for the calculation of S matrix. E. Stueckelberg, A. Petermann and W. Güttinger associated these indeterminateness with the so-called "ambiguities of current field theory" and introduced renormalization constants.

In this paper we study mainly some concrete examples which is important in quantum field theory such as $\delta\cdot\delta$, $\delta\cdot x^{-1}$ etc. and show also that the Güttinger's product is contained as a special case of our multiplication.

2. Notations and Definitions [8]. Let $\boldsymbol{Q}$ denote the set of all sequences $\{\varphi_n\}$ of functions $\varphi_n\in\mathcal{E}$, i.e. $\boldsymbol{Q}=\{\{\varphi_n\};\ \varphi_n\in\mathcal{E}\}$. Let $\widetilde{\boldsymbol{Q}}_\tau$ denote the set of all convergent sequences in τ topology. Let $\mathfrak{O}_\tau$ denote the set of all sequences which converge to zero in τ topology. Let Q_τ denote the set of classes such that $Q_\tau\equiv\boldsymbol{Q}/\mathfrak{o}_\tau=\{\mathfrak{c}(\ |\tau|\,\widetilde{T}_\alpha),\mathfrak{c}(\ |\tau|\infty_\beta)\}$. Let $\widetilde{Q}_\tau$ be the set of all convergent classes, i.e. $\widetilde{Q}_\tau\equiv\widetilde{\boldsymbol{Q}}_\tau/\mathfrak{o}_\tau=\{\mathfrak{c}(\ |\tau|\,\widetilde{T}_\alpha)\}$.

Let $\boldsymbol{Q}^{D'}$ be the set of all convergent (in D' topology) sequences $\{\varphi_n\}$, $\varphi_n\in\mathcal{E}$. Let $Q_\tau^{D'}$ be the set of all classes $Q_\tau^{D'}\equiv\boldsymbol{Q}^{D'}/\mathfrak{o}_\tau=\{\mathfrak{c}(T\,|\tau|\,\widetilde{T}_\alpha),\mathfrak{c}(T\,|\tau|\infty_\beta)\}$, where $\varphi_n\in\mathfrak{c}(T\,|\tau|\,\widetilde{T}_\alpha)$ means $\varphi_n\to T$ in $\mathfrak{D}'$, $\varphi_n\to\widetilde{T}_\alpha$ in τ.