71. Relations among Topologies on Riemann Surfaces. I

By Zenjiro KURAMOCHI

Mathematical Institute, Hokkaido University

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Let R be a Riemann surface and let $R_n(n=0,1,2,\cdots)$ be its exhaustion with compact relative boundary ∂R_n. Suppose R is a Riemann surface with positive boundary (if R has null-boundary, consider $R-R_0$ instead of R). Then we can introduce some topologies from the original topology (defined by local parameters) which are homeomorphic to the original topology in R. We know Stoilow's, Green's, K-Martin's and N-Martin's topologies[1] (we abbreviate them by $S.T$, $G.T$, $KM.\ T$ and $NM.\ T$ respectively in the present papers). Also we can define the ideal boundary B by the completion of R with respect to $\alpha\,(\alpha=S,\ G,\ KM$ or $NM)$-topology. When R is a subdomain in the z-plane, the boundary of R is realized. In this case also we can use the topology defined by Euclidean metric abbreviated by $E.T$. To study potential, analytic functions and the structure of Riemann surfaces, we use suitable topologies on R. But it is important to consider the relations among topologies on R.

Let $[p]^\alpha$ be a point of $\overline{R}=R+B$ with respect to α-topology and let $[v_n(p)]^\alpha=E\left[z\in\overline{R}: \overset{\alpha}{\mathrm{dist}}\,(p,z)<\dfrac{1}{n}\right]$, where $\overset{\alpha}{\mathrm{dist}}\,(p,z)$ is the distance between p and z with respect to α-topology. Suppose α and β-topologies are defined on $\overline{R}$. Then $\lim_n[\overline{v_n(p)}]^\alpha=p=\lim_n[\overline{v_n(p)}]^\beta$ for $p\in R$. If $\lim_n[\overline{v_n(p)}]^\alpha=[p]^\beta$ for every $p\in\overline{R}$, we say that α is finer than β and denote it by $\alpha\succ\beta$. If α is not finer than β and also β is not finer than α, we say that α and β are independent and denote it by $\alpha\succ\!\!\prec\beta$. Suppose $KM.\ T$ and $NM.\ T$ are defined in $\overline{R}$. Let B_1^γ be the set of γ-minimal point ($\gamma=K$ or N).[2] Then $B-B_1^\gamma=B_0^\gamma$ is an F_σ set of harmonic measure zero for K and of capacity zero for N respectively. Let G be a domain in R and $p\in B_1^\gamma$. If $K_{CG}(z,p)<K(z,p)({}_{CG}N(z,p)<N(z,p))$, we say $G\overset{K}{\ni}p(G\overset{N}{\ni}p)$, where $K_{CG}(z,p)({}_{CG}(N(z,p))$ is the least positive super ($\overline{\text{super}}$[3]) harmonic function in R (in $R-R_0$) larger than G. Then we proved that such domains have almost the

1) Z. Kuramochi: On the behaviour of analytic functions on the ideal boundary. II, Proc. Japan Acad., **38**, 188-193 (1962).

2) Z. Kuramochi: On potentials on Riemann surfaces, Journ. Hokkaido Univ., (1962).

3) See 2).