66. A Note on the First Boundary Value Problem on Martin Spaces Induced by Markoff Chains

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1. The first boundary value problem on Martin spaces induced by Markoff chains has been studied by J. L. Doob [1], T. Watanabe [6] and G. A. Hunt [4]. Their arguments are based on the probabilistic interpretation of the theory of R. S. Martin and the martingale convergence theorem established by J. L. Doob. In this paper, we state simple remarks for the conditions that any boundary point of the Martin space induced by a Markoff chain is regular with respect to the first boundary value problem. Our argument depends on the properties of sojourn solutions studied by W. Feller [2], but not on the martingale convergence theorem. The author wishes to express his thanks to Prof. S. Tsurumi, Mr. K. Sato and Prof. T. Watanabe for their valuable advice.

2. Let $x(t)$ be a temporally homogeneous Markoff chain on a countable set S with continuous time. We suppose that $x(t)$ is minimal in the sense of [3] and all points of S are transient and we consider a fixed reference measure γ, introduced by G. A. Hunt [4]. We concern with the Martin space $M(=S+\partial S)$, with its topology ρ, induced by $x(t)$ with respect to γ as in H. Kunita and T. Watanabe [5]. We shall denote by $B(\partial s)$ the Borel field consisting of all P-Borel subsets in ∂S.

Let $\mathrm{P}(x, \cdot)$ be the probability with the initial mass at $x \in S$ and let $z(i-0)=\lim_{n\to+\infty} z(n)$, where $z(n)$ is the n-th jumping time, then the harmonic measure $h(x, B)$ from x to $B \in B(\partial S)$ is, in our case,

$$h(x, B)=P(x, x(z(i-0)) \in B).$$

For any set $A \subset S$,

$$S_A(x)=P(x, \bigcup_{k\geq 1}\bigcap_{n\geq k}(x(z(n))\in A))$$

is called a sojourn solution and A a sojourn set, if $S_A(\gamma)>0$. The following Lemma 1 shows the close relation between harmonic measures and sojourn solutions.

We introduce here some notations:

$(\partial S)_F=(b;\, b\in\partial S,\, \mathrm{h}(\gamma, G)>0$ for any open neighborhood G of $b.$).

For any $b \in \partial S$ and $\alpha, \beta>0$, let

$$\partial G_\alpha^b=(b';\, b'\in\partial S,\, \rho(b', b)\leq\alpha),$$
$$G_\alpha^b=(x;\, x\in S,\, \rho(b, x)\leq\alpha),$$