112. On an Isomorphism of Galois Cohomology Groups $H^m(G, O_K)$ of Integers in an Algebraic Number Field

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Introduction. In my paper [1][1)] we proved the following theorem:

Let K be a normal extension over the rational number field $\boldsymbol{Q}$, and k be a subfield of K such that K/k is cyclic of prime degree p and that $k/\boldsymbol{Q}$ is normal of degree n. Then, for every dimension m the Galois cohomology group $H^m(G, O_K)$ $(G=G(K/k))$ of O_K with respect to K/k is isomorphic to the ns/e-ple direct sum of cyclic group of order p:

$$H^m(G, O_K)\cong\{\overbrace{p, p,\cdots, p}^{ns/e}\}.$$

There we proved this Theorem by showing that the 1-dimensional Galois cohomology group $H^1(G, O_K)$ of O_K with respect to K/k is isomorphic to the 0-dimensional Galois cohomology group $H^0(G, O_K)$ of O_K with respect to K/k:

$$H^1(G, O_K)\cong H^0(G, O_K).$$

In the present paper, we shall give another proof of this Theorem by showing that the 0-dimensional Galois cohomology group $H^0(G, O_K)$ of O_K with respect to K/k is isomorphic to the -1-dimensional Galois cohomology group $H^{-1}(G, O_K)$ of O_K with respect to K/k:

$$H^0(G, O_K)\cong H^{-1}(G, O_K).$$

Theorem. *Let K be a normal extension over the rational number field $\boldsymbol{Q}$, and k be a subfield of K such that K/k is cyclic of prime degree p and that $k/\boldsymbol{Q}$ is normal of degree n. Denote by G the Galois group of K/k, by O_K and O_k the rings of all integers in K and k respectively. Further, let v be the common ramification number with respect to K/k of all the prime divisors $\mathfrak{P}_i$ of p in K,*[2)] *and e be the common ramification order with respect to $k/\boldsymbol{Q}$ of all the prime divisors $\mathfrak{p}_i$ of p in k. Put $s=v-\left[\frac{v}{p}\right]\geqq 0$, where $[x]$ means Gaussian symbol.*

Then the -1-dimensional Galois cohomology group $H^{-1}(G, O_K)$ of O_K with respect to K/k is isomorphic to the ns/e-ple direct sum of cyclic group of order p:

1) Cf. H. Yokoi [1], Theorem 3.

2) Here we understand the ramification number v in the same way as we understood in [1]. Cf. H. Yokoi [1], [Remark].