110. On Cohomological Dimension for Paracompact Spaces. I

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1. **Introduction.** In 1954 H. Cohen [1] defined a cohomological dimension for locally compact Hausdorff spaces and proved several properties which are desirable for a dimension function. His definition reads as follows: a locally compact Hausdorff space X has $cdX \leqq n$ if and only if for each closed set C of X and for each compact set K in X $H^m(K \frown C; G)$ is the image of $H^m(K; G)$ of the homomorphism induced by the inclusion of $K \frown C$ into K where m is any integer such that $m \geqq n$ and $H^m(K; G)$ and $H^m(K \frown C; G)$ are the m-th Čech cohomology groups with the non-zero additive Abelian group G as coefficient.

In this paper we will modify his definition for paracompact Hausdorff spaces and establish some properties: the monotone property and the sum theorem.

All spaces in the present paper will be assumed to be paracompact Hausdorff unless otherwise specified, and all coefficient groups will be assumed to be non-zero additive Abelian groups.

2. **Preliminaries.** *Notations.* Let X be a space and let A be a closed set of X. For each non-negative integer n $H^n(X; G)$ means the n-th Čech cohomology group of X with G as coefficient and $H^n(X, A; G)$ means the n-th Čech cohomology group of X relative to A. If e is an element of $H^n(X; G)$, then we denote by $e|A$ the image of e by the homomorphism of $H^n(X; G)$ into $H^n(A; G)$ induced by inclusion.

Let α, β be open coverings of X. Since X is paracompact, all open coverings are assumed to be locally finite. For each open covering α of X we let $N(\alpha)$ be the nerve of α. If β is a refinement of α, then there is a projection Π of $N(\beta)$ into $N(\alpha)$ and this Π induces the homomorphisms $\Pi_{\alpha\beta}$ of n-cochain group, n-cocycle group and n-cohomology group of $N(\alpha)$ into n-cochain group, n-cocycle group and n-cohomology group of $N(\beta)$, respectively.

If $t^n = (U_0, \cdots, U_n)$ is an n-simplex of $N(\alpha)$, then we denote by $(t^n)_0$ the set $\bigcap_{k=0}^{n} U_k$. If $c^n = \sum_{\mu} a_\mu t^n_\mu$ is an n-cochain of $N(\alpha)$ (where each a_μ is a non-zero element of G), then we denote by $(c^n)_0$ the set