106. A Note on the Cut Extension of C-Spaces

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§1. Let R be *a semi-ordered linear space* which is *Archimedean*.[1] A semi-ordered linear space $\hat{R}$ is called the *cut extension of* R, if there exists a mapping of R into $\hat{R}$ ($R \ni a \to a^{\hat{R}} \in \hat{R}$) such that

(C. 1) $(\alpha a+\beta b)^{\hat{R}}=\alpha a^{\hat{R}}+\beta b^{\hat{R}}$ *for any* $a, b \in R$ *and real numbers* α, β;

(C. 2) $a \leqq b$ *if and only if* $a^{\hat{R}} \leqq b^{\hat{R}}$;

(C. 3) $\bigcap_{\lambda \in \Lambda} a_\lambda=0$ $(a_\lambda \in R, \lambda \in \Lambda)$ *implies* $\bigcap_{\lambda \in \Lambda} a_\lambda^{\hat{R}}=0$ *in* $\hat{R}$;

(C. 4) $\hat{R}$ *is universally continuous*;[2]

(C. 5) *for each* $\hat{a} \in \hat{R}$ *there exists a system of elements* $a_\lambda \in R$ $(\lambda \in \Lambda)$ *such that* $\hat{a}=\bigcup_{\lambda \in \Lambda} a_\lambda^{\hat{R}}$.

When we consider R as a lattice, $\hat{R}$: the *cut extension* of lattice R is nothing but a *normal completion* of R in Birkhoff's terminology [1].

It is well known ([4], Theorems 30.2 and 30.3) that *for any Archimedean semi-ordered linear space R there exists always $\hat{R}$: the cut extension of R, and $\hat{R}$ is determined uniquely up to an isomorphism.*

Now let E be a *compact Hausdorff space* throughout this paper and $C(E)$ be the space of all continuous functions defined on E. $C(E)$ is a semi-ordered linear space (by the usual addition and order) which is not always continuous, but Archimedean [2, 5, 6]. Thus, as is shown above, $\widehat{C(E)}$: the *cut extension* of $C(E)$ may be considered. The structure of $\widehat{C(E)}$ was investigated in [2] and it was proved that $\widehat{C(E)}$ *is isomorphic to the C-space* $C(\mathcal{E})$, *where* $\mathcal{E}$ *is the Boolean space associated with the lattice of regularly open sets*[3] *in* E, while $\mathcal{E}$ comes to be different from the original space E in most cases.

The aim of this note is to construct a function space on E which is isomorphic to $\widehat{C(E)}$. The result is the following:

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1) R is called *Archimedean*, if $\bigcap_{\nu=1}^{\infty} \frac{1}{\nu} a=0$ for every $0 \leqq a \in R$.

2) A semi-ordered linear space is called *universally continuous*, if for any bounded system of elements : $\{a_\lambda : a_\lambda \leqq a, \lambda \in \Lambda\}$ there exists $\bigcup_{\lambda \in \Lambda} a_\lambda$.

3) A subset G of E is called to be *regularly open*, if $G^{-o}=G$.