102. *Some Applications of the Functional-Representations of Normal Operators in Hilbert Spaces. II*

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Let $\{\lambda_\nu\}$, $S(\lambda)$, $\Phi(\lambda)$, and $R(\lambda)$ be the same notations as those defined in the statement of Theorem 1 [3] respectively, and $\Psi(\lambda)$ the second principal part of $S(\lambda)$ in the case where all the accumulation points of $\{\lambda_\nu\}$ form an uncountable set.

Since, by Theorem 1,

$$\frac{1}{2\pi i}\int_{|\lambda|=\rho}\frac{S(\lambda)}{(\lambda-z)^{k+1}}d\lambda=\frac{R^{(k)}(z)}{k!}\quad (k=0,1,2,3\cdots)$$

for every point z in the interior of the circle $|\lambda|=\rho$ with $\sup_\nu|\lambda_\nu|<\rho<\infty$, we obtain

$$\frac{1}{2\pi}\int_0^{2\pi}\frac{S(\rho e^{it})}{(\rho e^{it})^k}dt=\frac{R^{(k)}(0)}{k!}.$$

Consequently $R(\lambda)$ is expansible, on the domain $\{\lambda:|\lambda|<\infty\}$, in terms of integrals concerning the given function $S(\lambda)$ itself.

In this paper I have mainly two purposes: one is to find the expressions of $\Phi(\lambda)$ and $\Psi(\lambda)$ in terms of integrals concerning $S(\lambda)$ itself respectively, the other is to establish the relation between the maximum-modulus of $S(\lambda)$ on the circle $|\lambda-c|=\rho_1$ containing $\{\lambda_\nu\}$ and all the accumulation points of $\{\lambda_\nu\}$ inside itself and that of $R(\lambda)$ on the circle $|\lambda-c|=\rho_2$ with $\rho_2<\rho_1$.

Theorem 4. If the set of all the accumulation points of $\{\lambda_\nu\}$ is uncountable, then the second principal part $\Psi(\lambda)$ of $S(\lambda)$ in Theorem 1 is expressible in the form

$$(1)\qquad \Psi\Big(\frac{\rho e^{i\theta}}{\kappa}\Big)=\frac{1}{2\pi}\int_0^{2\pi}S(\rho e^{it})\frac{1-\kappa^2}{1+\kappa^2-2\kappa\cos(\theta-t)}dt-\frac{1}{2\pi}\int_0^{2\pi}S(\rho e^{it})\frac{e^{it}}{e^{it}-\kappa e^{i\theta}}dt-\sum_{\alpha=1}^{m}\sum_\nu c_\alpha^{(\nu)}\Big(\frac{\rho e^{i\theta}}{\kappa}-\lambda_\nu\Big)^{-\alpha}$$

for every κ with $0<\kappa<1$ and every ρ with $\sup_\nu|\lambda_\nu|<\rho<\infty$; and if, contrary to this, the set of all the accumulation points of $\{\lambda_\nu\}$ is countable, then

$$(2)\qquad \sum_{\alpha=1}^{m}\sum_\nu c_\alpha^{(\nu)}\Big(\frac{\rho e^{i\theta}}{\kappa}-\lambda_\nu\Big)^{-\alpha}=\frac{1}{2\pi}\int_0^{2\pi}S(\rho e^{it})\frac{1-\kappa^2}{1+\kappa^2-2\kappa\cos(\theta-t)}dt-\frac{1}{2\pi}\int_0^{2\pi}S(\rho e^{it})\frac{e^{it}}{e^{it}-\kappa e^{i\theta}}dt$$