101. Open Basis and Continuous Mappings. II*)

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(Comm. by K. KUNUGI, M.J.A., Oct. 12, 1962)

Let X and Y be T_1-spaces and let $f(X)=Y$ be a continuous mapping. f is said to be an S-mapping if the inverse image $f^{-1}(y)$ is separable[1] for each point y of Y. By the *open S-image*, we mean the image of an open continuous S-mapping. V. I. Ponomarev [4] has recently obtained the following theorem: *a T_1-space X has a point-countable open base if and only if X is an open S-image of a 0-dimensional metric space.*

In this note, we shall obtain an analogous theorem concernig the locally countable (star-countale) open base and we shall next investigate the open base of the inverse image space of an open continuous S-mapping.

1. We begin with proving the following theorem which is analogous to V. I. Ponomarev's theorem.

Theorem 1. *A T_1-space X has a locally countable (star-countable) open base if and only if X is an open S-image of a locally separable 0-dimensional metric space.*

Proof. As the "if" part is easily seen from our previous note ([1], Theorem 10, Remark 3), we shall prove the "only if" part. Since it is easily verified that X has a star-countable open base if and only if X has a locally countable open base, we deal with the case of the star-countable open base. Let X have a star-countable open base $\mathfrak{A}=\{U_\alpha\}$, then X is decomposed in such a way that $X=\bigcup_{\gamma\in\Gamma}A_\gamma$, $A_\gamma=\bigcup\{U_\alpha\in\mathfrak{A}_\gamma\}$, $A_\gamma\cap A_{\gamma'}=\phi$ for $\gamma\neq\gamma'$, $\gamma,\gamma'\in\Gamma$ where each $\mathfrak{A}_\gamma$ is a countable subfamily of $\mathfrak{A}$ [2, 6]. Then each A_γ has a countable open base $\mathfrak{A}_\gamma$ for each $\gamma\in\Gamma$. Let $\mathfrak{A}_\gamma=\{U_n^{(\gamma)}\mid n=1,2,\cdots\}$. For every point x of A_γ $\{U_n^{(\gamma)}\mid x\in U_n^{(\gamma)}, U_n^{(\gamma)}\in\mathfrak{A}_\gamma\}$ is countable. Let us denote this collection by $\{U_{n_i(x)}^{(\gamma)}\mid i=1,2,\cdots\}$, then, since X is a T_1-space, we have $\bigcap_{i=1}^{\infty}U_{n_i(x)}^{(\gamma)}=x$. If the intersection of all sets belonging to a countable subfamily $\{U_{n_i}^{(\gamma)}\}$ of $\mathfrak{A}_\gamma$ is a single point, then we define $\xi=(n_1,n_2,\cdots)$. Now let B_γ denote the set of all such ξ. We can define the topology

*) This note is a continuation of our previous note [1].

1) A set A is said to be separable when there exists a countable subset B of A such that $\bar{B}\supset A$. By the definition due to V. I. Ponomarev, S-mapping means the continuous mapping such that the inverse image $f^{-1}(y)$ is perfectly separable for each point y of Y, but we define here in the weaker sense than this.