99. On the Normalizer of an f-Ring

By Barron BRAINERD

Institute of Advanced Studies, Australian National University, Canberra, Australia

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1. *Introduction.* D. G. Johnson has studied the structure of f-rings, that is, lattice ordered rings R which satisfy the following condition:

For $a, b, c \in R$, if $c \geq 0$ and $a \frown b = 0$, then $ca \frown b = ac \frown b = 0$.

In [4] he shows that an f-ring without nilpotent elements can be embedded as a sub-f-ring of an f-ring with an identity. In [7] Henriksen and Isbell have shown that f-ring without left or right annihilators can also be embedded in an f-ring with identity. In this note we intend to prove following theorem which improves these results.

THEOREM 3.1. *If R is an f-ring without non-zero right or left annihilators, then the normalizer of R is an f-ring which is, of course, without non-zero right or left annihilators.*

The methods employed in this note are quite different from those used by either Johnson or Henriksen and Isbell.

Recall that a *left annihilator* of ring R is an element $a \in R$ with the property: $aR = 0$. A right annihilator is defined analogously.

R. E. Johnson in [5] discusses the normalizer $N^l(R)$ of a ring R without non-zero left annihilators. This ring is composed of those mappings φ of R into R which satisfy the following conditions:

(N 1) $\varphi(a+b) = \varphi(a) + \varphi(b)$ for $a, b, \in R$.

(N 2) $\varphi(ab) = \varphi(a)b$ for $a, b, \in R$.

(N 3) For each $a \in R$, there is an element $(a)\rho_\varphi \in R$ such that $a\varphi(x) = (a)\rho_\varphi x$ for every $x \in R$.

The conditions (N 1) and (N 2) indicate that $N^l(R)$ is a subset of the ring $E^l(R)$ of R-endomorphisms of R considered as a left R-module. The set $N^l(R)$ is closed with respect to the ring operations of $E^l(R)$ and hence is a sub-ring of $E^l(R)$. The set $N^l(R)$, endowed with the operations of $E^l(R)$, we shall call the *normalizer of* R.

Since 0 is the only left annihilator of R in R, the mapping $a \to T_a$ is an isomorphism of R into $N^l(R)$ provided T_a is defined to be the mapping $T_a : x \to ax$.

In [5], R. E. Johnson shows that if the ring R is an ideal of a ring S and if the only left annihilator of R in S is zero, then S is isomorphic to a subring of $N^l(R)$. In addition, one easily verifies