# 98. The Product of a Logarithmic Method and the Sequence-to-Sequence Quasi-Hausdorff Method

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1. *Definitions.* $\mu_n$, defined by

(1.1) $$\mu_n=\int_0^1 t^n d\chi(t) \quad (n=0,1,2,\cdots),^*$$

where $\chi(t)$ is a real function of bounded variation in $(0,1)$, is called the *moment constant* of *rank* $n$ generated by the *mass-function* $\chi(t)$. If, further,

(1.2) $$\chi(1)=1,\ \chi(+0)=\chi(0)=0,^+$$

$\mu_n$ is said to be a *regular* moment constant.
The matrix $\lambda\equiv(H,\mu_n)$, defined by

(1.3) $$\lambda_{nk}=\begin{cases}\binom{n}{k}\Delta^{n-k}\mu_k & (n\geq k)\\ 0 & (n<k),\end{cases}$$

is termed the *Hausdorff-matrix* corresponding to the sequence of moment constants $\{\mu_n\}$. The summability $(H,\mu_n)$ of a sequence $\{s_n\}$ to the sum $s$ is defined as the convergence to a finite limit $s$ of its *Hausdorff transform*, or simply $(H,\mu_n)$ transform, $\sigma_n$, where

(1.4) $$\sigma_n=\sum_{k=0}^{n}\lambda_{nk}\,s_k \quad (n=0,1,2,\cdots).$$

The transpose of the Hausdorff matrix, that is, the matrix $\lambda^*\equiv(H^*,\mu_n)$, defined by

(1.5) $$\lambda^*_{nk}=\begin{cases}\binom{k}{n}\Delta^{k-n}\mu_n & (n\leq k)\\ 0 & (n>k)\end{cases}$$

is termed the *Quasi-Hausdorff matrix* corresponding to the sequence of moment constants $\{\mu_n\}$.

The sequence-to-sequence *Quasi-Hausdorff transform*, or simply the $(H^*,\mu_n)$ transform, $\sigma_n{}^*$ of a sequence $\{s_n\}$ is defined by

(1.6) $$\sigma_n{}^*=\sum_{k=n}^{\infty}\binom{k}{n}\Delta^{k-n}\mu_n\,s_k.$$

Since $\mu_n$ is given by (1.1), we also have

(1.7) $$\sigma_n{}^*=\sum_{k=n}^{\infty}\int_0^1 s_k\binom{k}{n}t^n(1-t)^{k-n}d\chi(t).$$

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* The function $t^0$ is defined at $t=0$ so as to be continuous; thus

$$\mu_0=\int_0^1 d\chi(t).$$

+ The assumption $\chi(0)=0$ is not a substantial restriction.