96. On Spherical Functions over $\mathfrak{p}$-adic Fields

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A $\mathfrak{p}$-adic analogue of the spherical function was first considered by Mautner[1] in the case of PL_2 and then by Tamagawa[2] for GL_n over any $\mathfrak{p}$-adic division algebra. The purpose of the present note is to show that the main part of the theory holds more generally under certain conditions, which are satisfied by almost all classical simple groups. It happened that similar considerations were contained in an independent work of Bruhat.[3]

1. *Notations and Assumptions.* Let k be a $\mathfrak{p}$-adic number field. We denote by $\mathfrak{o}$ and $\mathfrak{p}=(\pi)$ the valuation ring in k and its unique prime ideal, respectively, π denoting a prime element. Let G be an algebraic subgroup of $GL_n(k)$, defined over k, and set as follows:

A= the connected component of the identity (in the sense of Zariski topology) of the subgroup of G consisting of all diagonal matrices in G,

N= the subgroup of G consisting of all upper unipotent matrices (i.e. matrices $x=(x_{ij})$ such that $x_{ii}=1$, $x_{ij}=0$ $(i>j)$) in G,

U= the subgroup of G consisting of all $\mathfrak{o}$-unit matrices (i.e. matrices $x=(x_{ij})$ such that $x_{ij}\in\mathfrak{o}$ and $\det x\notin\mathfrak{p}$) in G.

In the following, we assume that G satisfies the following two conditions (I), (II).

(I) $$G=U\cdot AN=U\cdot A\cdot U.$$

This implies that U is a maximal (open) compact subgroup of G (viewed as a topological group in the $\mathfrak{p}$-adic topology), and that A, N are, respectively, a maximal trivial torus and a maximal unipotent subgroup of G (viewed as a linear algebraic group over k). Moreover, it follows from the existence of maximal compact subgroup that G is reductive and hence unimodular. We normalize the (both side invariant) Haar measure dx on G in such a way that $\int_U dx=1$.

Now, for $(m)=(m_1,\cdots,m_n)\in\boldsymbol{Z}^n$ ($\boldsymbol{Z}$= ring of rational integers), put $\pi^{(m)}=\mathrm{diag.}(\pi^{m_1},\cdots,\pi^{m_n})$ and call A_π the subgroup of A consisting of all matrices of this form in A. If $\dim A=\nu$, we have $A\cong(k^*)^\nu$, k^* denoting the multiplicative group of non-zero elements in k, so that there exists a lattice M of rank ν in $\boldsymbol{Z}^n$ such that $A_\pi=\{\pi^{(m)}\mid(m)\in M\}$. For $u\in U$, normalizing A, we have

$$u\pi^{(m)}u^{-1}=\pi^{w(m)} \quad \text{for} \quad (m)\in M,$$