149. On the Apriori Estimate for the Solution of Some Semi-Linear Wave Equation for Higher Space Dimension

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We have considered [1] already the equation of the following type;

$$\Delta u = u_{tt} + g(u) \tag{1}$$

and obtained an apriori estimate for the solution of the Cauchy problem for 3 space dimension under the condition:

$$\begin{aligned} &\text{i)}\quad G(u)=\int_0^u g(u)du \geqq -L \\ &\text{ii)}\quad |g'(u)| \leqq c|u|^2 \quad (|u|\geqq k)\ c \text{ is a constant.} \end{aligned} \tag{2}$$

In this paper, we shall obtain the analogous results for the space dimension n higher than 3 assuming that the solution belongs to the space $D_{L^2}^{[\frac{n}{2}]+2}$. Our conditions for these cases are the following;

$$\begin{aligned} &\text{i)}\quad G(u)=\int_0^u g(u) \geqq -L \quad (L>0) \\ &\text{ii)}\quad |g'(u)| \leqq c|u|^{\frac{2}{n-2}} \quad 2<n\leqq 6 \quad (|u|\geqq k) \\ &\text{iii)}\quad |g''(u)| \leqq M, \quad |g'''(u)| \leqq M_1. \end{aligned} \tag{3}$$

At first we introduce new unknown functions and we obtain a system of equations (4).

$$\begin{aligned} &\frac{\partial u}{\partial t} = v \qquad \left(p_i = \frac{\partial u}{\partial x_i}\right) \\ &\frac{\partial v}{\partial t} = \sum_{i=1}^n p_{ix_i} - g(u) \\ &\frac{\partial p_i}{\partial t} = \frac{\partial v}{\partial x_i} \quad (i=1,2,\cdots,n). \end{aligned} \tag{4}$$

Our initial conditions for the Cauchy problem for u, v, p_i are $u(x,0)$, $v(x,0)$, $p_i(x,0) \in C_{0x}^{[\frac{n}{2}]+2}$, $C_{0x}^{[\frac{n}{2}]+1}$, $C_{0x}^{[\frac{n}{2}]+1}$ respectively and we suppose $g(u) \in C_u^3$; here we denote C_0^n the function space of the functions with continuous derivatives of n th order and with compact supports.

If we introduce an energy $E_0(t)$ of the solution u by the following integral form, we can easily prove its conservation, that is to say

$$E_0(t) = \int \left[G(u) + \frac{1}{2}v^2 + \frac{1}{2}\sum_{i=1}^n p_i^2 \right] dx \tag{5}$$