148. *Configuration Theorems in Affine Plane*

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The axiomatic theory of Euclidean geometry originated by D. Hilbert has led a series of configurational propositions. In 1943, M. Hall [3] has given a new method of coordinating a projective plane. By Hall method, many configurational propositions have been obtained by L. A. Skorniakov [4], B. I. Argunov [1]. In his interesting article [2], R. H. Bruck has given the foundation of an affine plane by Hall method. In this paper, we shall consider configurational propositions in an affine plane.

For the completeness, we shall give some definitions briefly. An *affine plane* π is a set of points and lines among which a relation on incidence satisfies the following conditions:

1. *Two distinct points are incident with one and only one line.*
2. *Two distinct lines are incident with at most one point.*
3. *Through a point exterior to a given line there is exactly one line having no intersection point with the given line.*
4. *There exist four points in general position* (no three of four points are incident with one line).

If there is no point on two lines a and b, then we call a and b parallel and write $a//b$. As well known, $a//b$ and $b//c$ implies $a//c$. Every line has same number of points, every point lies on same number of lines.

Let X, Y, O, E be four points in general position on π. We call the point O the origin of coordinates, line OE the unit line. Let $\boldsymbol{M}$ be the set of symbols which is 1–1 correspondence with the point set of the line OE. Let 0(1) by the element of $\boldsymbol{M}$ with corresponds with $O(E)$. We denote by $a, b, c, \cdots$ the element of $\boldsymbol{M}$. By the well-known method (see R. H. Bruck [2]), we can define a ternary ring R on π. We may introduce coordinates in an affine plane π in the following way (see Figs. 1, 2).

We take three distinct lines OA, OB, and OE through O which we call the *x-axis*, the *y-axis* and the *unit line*. To each point A of OE, we assign a coordinate $A(a, a)$, $a \in \boldsymbol{M}$. For any point P, we define its coordinate $P(a, b)$ as Fig. 1, where $OY//AP$, $OX//BP$. If the line parallel to the y-axis is incident with a given point $P(a, b)$, the equation of the line is given by $x=a$. Suppose that a line l intersects the y-axis, and let l' be through the origin parallel to the line