146. *Homotopy Groups with Coefficients and a Generalization of Dold-Thom's Isomorphism Theorem. II*

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This paper is a continuation of the previous paper I. Notations and definitions of the paper I will be used without any comment.

3. **Infinite symmetric products.** Let X be a Hausdorff space with base point o. The q-fold symmetric product $SP^q(X)$ is a space obtained from the topological product $X^q=X\times\cdots\times X$ (q-fold) by identifying all points which differ only in the order of the components. The image of $(x_1,\cdots,x_q)\in X^q$ by the identification map is denoted by $[x_1,\cdots,x_q]\in SP^q(X)$. We define the inclusion map i_q: $SP^q(X)\to SP^{q+1}(X)$ by $i_q[x_1,\cdots,x_q]=[o,x_1,\cdots,x_q]$. The infinite symmetric product of X with respect to the base point o is the inductive limit of the sequence $X=SP^1(X)\xrightarrow{i_1}SP^2(X)\xrightarrow{i_2}\cdots$ and is denoted by $SP(X,o)$. $SP(X,o)$ is a Hausdorff space (cf. [1], p. 254).

A map $f:(X,o)\to(X',o')$ induces a map $f^{\Sigma q}: SP^q(X)\to SP^q(X')$ defined by $f^{\Sigma q}[x_1,\cdots,x_q]=[f(x_1),\cdots,f(x_q)]$. Clearly, $f^{\Sigma q}$ is compatible with the inclusion i_q. Then a map $f^\Sigma: SP(X,o)\to SP(X',o')$ can be defined by $f^\Sigma|SP^q(X)=f^{\Sigma q}$. Obviously, if f and g are homotopic, then f^Σ and g^Σ are homotopic. Hence the homotopy type of $SP(X,o)$ depends only on that of (X,o). It is easily verified that if A is closed (open) in X and $i: A\to X$ is the inclusion, then the induced maps $i^{\Sigma q}: SP^q(A)\to SP^q(X)$ and $i^\Sigma: SP(A,o)\to SP(X,o)$ are homeomorphisms into.

An addition $\mu: SP(X,o)\times SP(X,o)\to SP(X,o)$ can be defined by $\mu([x_1,\cdots,x_q],[y_1,\cdots,y_r])=[x_1,\cdots,x_q,y_1,\cdots,y_r]$. $SP(X,o)$ is a free abelian semi-group over X with o as the unit element with respect to the addition μ. μ is continuous on any subset of $SP^q(X)\times SP^r(X)$ and on any compact subset of $SP(X,o)\times SP(X,o)$.

Now let X be a CW-complex, $A\ni o$ a connected subcomplex of X, X/A a space obtained from X by contracting A to a point $\bar{o}$ (the base point of X/A), and let $p:(X,o)\to(X/A,\bar{o})$ be the identification map. Then the following is obtained in [1], §5.

Proposition 2. *Under the above assumptions the induced map* $p^\Sigma: SP(X,o)\to SP(X/A,\bar{o})$ *is a quasi-fibering*[1] *with a fiber* $(p^\Sigma)^{-1}(\bar{o})$

1) A map $p: E\to B$ is said to be a quasi-fibering if p is onto and induces an isomorphism $p_*:\pi_n(E,F,x)\approx\pi_n(B,b)$ for any $b\in B$, $x\in F=p^{-1}(B)$ and $n\geqq 0$.