145. Homotopy Groups with Coefficients and a Generalization of Dold-Thom's Isomorphism Theorem. I

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1. Introduction. A. Dold and R. Thom established in [1] the existence of the following natural isomorphism

$$H_q(X) \approx \pi_q(SP(X, o)), \quad q \geqq 1,$$

for a connected CW-complex X with base point o, where $SP(X, o)$ denotes the infinite symmetric product of X. Professor K. Morita conjectured that there exists a natural isomorphism

$$H_q(X; G) \approx \pi_q(SP(X, o); G), \quad q \geqq 3,$$

for the homotopy groups with coefficients (in a finitely generated abelian group G) in the sense of Katuta [2]. In [3] we have proved that there exists the isomorphism above when X is a 1-connected countable simplicial complex. Here we shall show that the conjecture is true when X is a 1-connected CW-complex. The following theorem which was obtained in our previous paper [4] will play an important role in our proof.

Theorem 1. *Let spaces $E \supset F$, $B \supset C$ and a map $p:(E, F) \rightarrow (B, C)$ be given. If p is a weak homotopy equivalence of pairs of spaces, i.e. if p induces an isomorphism*

$$p_* : \pi_n(E, F) \approx \pi_n(B, C) \quad \text{for any } n \geqq 0,$$

then for a CW-complex K the induced map $'p:(E^K, F^K) \rightarrow (B^K, C^K)$ is a weak homotopy equivalence of pairs of mapping spaces, i.e. $'p$ induces an isomorphism

$$'p_* : \pi_n(E^K, F^K) \approx \pi_n(B^K, C^K) \quad \text{for any } n \geqq 0$$

where we mean a 1–1 correspondence by an isomorphism if $n \leqq 1$.

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2. Homotopy groups with coefficients. Throughout this paper we consider only spaces with base point and maps carrying the base point to the base point. Let G be a finitely generated abelian group. Y. Katuta defined homotopy groups with coefficients in G, $\pi_q(X; G)$, for $q \geqq 3$ and each space X as follows. Let us consider S^1 the unit circle in the complex number plane with 1 as the base point and let $\rho_m: S^1 \rightarrow S^1$ be the map defined by $\rho_m(e^{i\theta}) = e^{im\theta}$, for a positive integer m. Let $\rho_m^q: S^q \rightarrow S^q$ be the $(q-1)$-fold suspension $S^{q-1}\rho_m$[1] of ρ_m. Then

1) The suspension $Sf: SX \rightarrow SY$ of a map $f: X \rightarrow Y$ is defined by $Sf(s, x) = (s, f(x))$ for $s \in S^1$ and $x \in X$, and the q-fold suspension of f by $S^q f = S(S^{q-1} f)$ (see also the foot note[3]).