143. Some Characterizations of $\mathfrak{m}$-paracompact Spaces. II

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In this paper we study some characterizations of $\mathfrak{m}$-paracompact and normal spaces in the form of the selection theorems.[1] Let X and Y be topological spaces. 2^Y will denote the family of non-empty subsets of Y. A function from a subset of X to 2^Y is called a *carrier*. If φ: $X\to 2^Y$, then a *selection* for φ is a continuous function $f: X\to Y$ such that $f(x)\in\varphi(x)$ for every $x\in X$. A carrier φ: $X\to 2^Y$ is *lower semi-continuous* if, whenever $V\subset Y$ is open in Y, $\{x\in X\mid\varphi(x)\cap V\neq\phi\}$ is open in X, where ϕ denotes the null set. For a Banach space or a complete metric space Y, we shall consider the following families of sets.

$$A(Y)=\{S\in 2^Y\mid S \text{ is closed}\},$$
$$K(Y)=\{S\in 2^Y\mid S \text{ is convex}\},$$
$$F(Y)=\{S\in K(Y)\mid S \text{ is closed}\},$$
$$C(Y)=\{S\in F(Y)\mid S \text{ is compact or } S=Y\}.$$

The following theorem seems to be interesting for us in the point of view that Michael's results [3, Theorems 3.1″ and 3.2″], which were separately stated and proved for paracompact spaces and countably paracompact spaces, are unified.

Theorem 1. *The following properties of a T_1-space are equivalent.*

(a) *X is $\mathfrak{m}$-paracompact and normal.*

(b) *If Y is a Banach space which has an open base of power $\leqq\mathfrak{m}$, then every lower semi-continuous carrier $\varphi: X\to F(Y)$ admits a selection.*

To prove this theorem, the following lemmas and Theorem 2 in the previous paper [2] are useful.

Lemma 1. *If X is $\mathfrak{m}$-paracompact and normal, Y a normed linear space with an open base of power $\leqq\mathfrak{m}$, $\varphi: X\to K(Y)$ a lower semi-continuous carrier, and if V is a convex neighborhood of the origin of Y, then there exists a continuous function $f: X\to Y$ such that $f(x)\in\varphi(x)+V$ for every x in X.*

Proof. Since $\{y-V\}_{y\in Y}$ is an open covering of Y and Y has an open base with power $\leqq\mathfrak{m}$, there exists a locally finite open refinement $\{W_\lambda\mid\lambda\in\Lambda\}$ of $\{y-V\}_{y\in Y}$ with $|\Lambda|\leqq\mathfrak{m}$. Let $U_\lambda=\{x\in X\mid\varphi(x)\cap W_\lambda$

1) Cf. E. Michael [3].