141. *Some Applications of the Functional-Representations of Normal Operators in Hilbert Spaces. III*

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In this paper we shall turn to the problem of finding the extended Fourier-series expansion corresponding to each of the functions $S(\lambda)$, $\Phi(\lambda), \Psi(\lambda)$, and $R(\lambda)$ defined in the statement of Theorem 1 [cf. Vol. 38, No. 6 (1962), pp. 263–268].

Theorem 6. Let $\{\lambda_\nu\}$, $S(\lambda)$, and $R(\lambda)$ be the same notations as those in Theorem 1 respectively. Then, for every ρ with $\sup_\nu|\lambda_\nu|<\rho<\infty$ and every κ with $0\leqq\kappa<\infty$,

$$(7) \qquad R(\kappa\rho e^{i\theta})=\frac{a_0}{2}+\frac{1}{2}\sum_{n=1}^{\infty}(a_n-ib_n)(\kappa e^{i\theta})^n \quad (\theta\text{: variable}),$$

where

$$(8) \qquad \begin{cases} a_n=\dfrac{1}{\pi}\displaystyle\int_0^{2\pi}S(\rho e^{it})\cos nt\,dt \\ b_n=\dfrac{1}{\pi}\displaystyle\int_0^{2\pi}S(\rho e^{it})\sin nt\,dt \end{cases} \qquad (n=0,1,2,3,\cdots)$$

and the series on the right-hand side converges absolutely and uniformly.

Proof. It follows from Theorem 1 that

$$\begin{aligned}\frac{1}{2}(a_n-ib_n)&=\frac{1}{2\pi}\int_0^{2\pi}S(\rho e^{it})e^{-int}dt \quad (n=0,1,2,3,\cdots)\\ &=\frac{1}{2\pi i}\int_{|\lambda|=\rho}\frac{S(\lambda)\rho^n}{\lambda^{n+1}}d\lambda\\ &=\frac{R^{(n)}(0)\rho^n}{n!},\end{aligned}$$

where $0!$ and $R^{(0)}(0)$ denote 1 and $R(0)$ respectively, so that

$$\begin{aligned}\frac{a_0}{2}+\frac{1}{2}\sum_{n=1}^{\infty}(a_n-ib_n)(\kappa e^{i\theta})^n&=\sum_{n=0}^{\infty}\frac{R^{(n)}(0)}{n!}(\kappa\rho e^{i\theta})^n \quad (0\leqq\kappa<\infty)\\ &=R(\kappa\rho e^{i\theta}).\end{aligned}$$

In addition, the absolute and uniform convergence of the series on the right-hand side of (7) is a direct consequence of the hypothesis that $R(\lambda)$ is regular on the domain $\{\lambda: |\lambda|<\infty\}$.

Theorem 7. Let $\{\lambda_\nu\}$, $S(\lambda)$, and $R(\lambda)$ be the same notations as before. Then, for every ρ with $\sup_\nu|\lambda_\nu|<\rho<\infty$ and every κ with $0<\kappa<1$,