139. *Unique Continuation Theorem of Elliptic Systems of Partial Differential Equations*

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1. Let L be a linear partial differential operator defind in a domain $\mathfrak{D}$ of the n-dimensional Euclidean space R^n. For simplicity, we assume that the origin 0 of R^n is contained in $\mathfrak{D}$. Denoting by $x=(x_1,\cdots,x_n)$ a point of R^n, we can write

$$L=L(x, D)=\sum_{\alpha} a_\alpha(x)D^\alpha,$$

where α is a sequence $(\alpha_1,\cdots,\alpha_n)$ of n non-negative integers,

$$D^\alpha=\frac{\partial^{\alpha_1}}{\partial x_1^{\alpha_1}}\cdots\frac{\partial^{\alpha_n}}{\partial x_n^{\alpha_n}}$$

and each $a_\alpha(x)$ is a real-valued continuous function in $\mathfrak{D}$. We use a notation $|\alpha|=\alpha_1+\cdots+\alpha_n$. Put $r=|x|=(\sum_{i=1}^{n} x_i^2)^{1/2}$.

Friedman [3] proved the following.

Let $u(x)$ be a constant-signed solution, in $\mathfrak{D}$, of an elliptic differential equation $Lu=0$ of order $2s$. If

$$(1)\qquad \lim_{r\to 0}\frac{D^\alpha u(x)}{r^k}=0 \quad \text{for any positive integer } k,$$

where α is an arbitrary sequence with $|\alpha|\leqq 2s-1$, then $u(x)$ vanishes identically in $\mathfrak{D}$.

And Pederson [4] gave an improvement of this theorem. That is, he proved that, in the above theorem, the assumption (1) can be replaced by the condition that there exists a positive integer N satisfying

$$\lim_{r\to 0}\frac{D^\alpha u}{r^N}=0$$

for every $\alpha(0\leqq|\alpha|\leqq 2s-1)$ and being dependent on L and independent of u.

Now consider an elliptic system of linear partial differential equations

$$(2)\qquad \sum_{j=1}^{p} l_{ij}u_j=0 \quad (i=1,\cdots,p)$$

in unknown functions $u_1,\cdots,u_p$, where l_{ij} is a linear partial differential operator with variable coefficients continuous in $\mathfrak{D}$. Carleman [1] proved the following.

In the case when in (2), $p=2, n=2$ and $l_{ij}(i,j=1,2)$ are of order 1, each solution u_j of (2), satisfying