136. 2-Primary Components of the Homotopy Groups of Spheres and Lie Groups

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In our previous papers [1], [2], we have stated some results on 2-primary components of the homotopy groups of spheres and some Lie Groups. In the present note, we should like to report some additional results on the same subjects, and correct some errors in the preceeding papers due to misprints or otherwise. The same notations as in [1], [2] will be used.

In [1], to (1.5) on page 184, we should like to add

$$[\iota_6, \iota_6] \in \{\nu_6, \eta_9, 2\iota_{10}\} \quad \mathrm{mod}\ 2\pi_{11}(S^6),$$

(I owe this remarks to professor Toda, to whom I express my sincere thanks for his interest in my papers and valuable discussion.) and to (2.6) on page 186,

$$[\iota_{13}, \iota_{13}] = \tau_{13}.$$

In [2], we have introduced the generators s^n_m, u^n_m, and r^n_m of the homotopy groups of Sp_n, U_n, and R_n, respectively. When I wrote [2], I did not know whether and how they could be represented in terms of other known elements, but I could show in the meantime that the following relations hold.

(1) $s^2_7 \in \{\sigma''^2_3, \alpha_3, 12\iota_6\}$ $\quad \mathrm{mod}\ 12\pi_7(Sp_2)$,

$s^3_{11} \in \{i''^{3,2}, T''_2, 5!\,\iota_{10}\}$ $\quad \mathrm{mod}\ 5!\,\pi_{11}(Sp_3)$.

(2) $u^3_5 \in \{\sigma'^3_3, \eta_3, 2\iota_4\}$ $\quad \mathrm{mod}\ 2\pi_5(U_3)$,

$u^3_{10} \in \{\sigma'^3_3, \eta_3, \nu_4 \circ \eta_7 \circ \eta_8\}$ $\quad \mathrm{mod}\ 0$,

$u^3_{11} \in \{\sigma'^3_3, \eta_3, \nu_4 \circ \nu_7\}$ $\quad \mathrm{mod}\ (\sigma'^3_3 \circ \varepsilon_3)$,

$u'^3_{12} \in \{u^3_5, 4\nu_5, \nu_8\}$ $\quad \mathrm{mod}\ (\sigma'^3_3 \circ \delta_3)$,

$u^4_7 \in \{i'^{4,3}, \sigma'^3_3 \circ \alpha_3, 6\iota_6\}$ $\quad \mathrm{mod}\ (6\pi_7(U_4)$,

$u^5_9 \in \{i'^{5,4}, T'_4, 24\iota_8\}$ $\quad \mathrm{mod}\ 24\pi_9(U_5)$,

$u^5_{12} \in \{i'^{5,4}, T'_4, 4\nu_8\}$ $\quad \mathrm{mod}\ (u'^5_{12})$,

$u^7_{13} \in \{i'^{7,6}, T'_6, 6!\,\iota_{12}\}$ $\quad \mathrm{mod}\ 6!\,\pi_{13}(U_7)$.

(3) $r^7_{11} \in \{i^{7,6}, 2r^6_8, \eta_8, 4\iota_9\}$ $\quad \mathrm{mod}\ 4\pi_{11}(R_7) + (r^7_8 \circ \nu_8)$,

$r^{11}_{12} \in \{i^{11,10}, T_{10}, \eta_9 \circ \eta_{10}\}$ $\quad \mathrm{mod}\ 0$.

The characteristic class T''_2 of Sp_3 and the characteristic class T'_4 of U_5 can be represented as follows:

(4) $T''_2 \in \{\sigma''^2_3, \alpha_3, \nu_6\}$ $\quad \mathrm{mod}\ 4\pi_{10}(Sp_2)$,

$T'_4 \in \{i'^{4,3}, \sigma'^3_3 \circ \alpha_3, \nu_6\}$ $\quad \mathrm{mod}\ (u^4_5 \circ \nu_5)$.

Using these relations (1)−(4), the following can be proved.

(5) $T''_2 \circ \eta_{10} = \sigma''^2_3 \circ \varepsilon_3$, $\quad s^2_7 \circ \nu_7 = 4\,T''_2$.