162. *An Extension of the Interpolation Theorem of Marcinkiewicz*

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§1. Introduction. In this paper we show that the Marcinkiewicz interpolation theorem of operators (e.g. see Zygmund [5]) holds good for Hardy class H_p or class $\mathfrak{H}_p$ introduced by Stein-Weiss [4].

H_p-class ($p>0$) is the space of all functions analytic in the unit circle such that

$$||\varphi||_p=\lim_{r\to 1}\left\{\frac{1}{2\pi}\int_{-\pi}^{\pi}|\varphi(re^{i\theta})|^p d\theta\right\}^{1/p}$$

is finite. $\mathfrak{H}_p$-class is the space of the vectors $F(X,y)=(u(X,y), v_1(X,y),\cdots,v_n(X,y))$ whose components are all harmonic in half-space $E^+_{n+1}=\{(X,y);X\in E_n, y>0\}$[1] and satisfy the generalized Cauchy-Riemann equations,

$$\frac{\partial u}{\partial y}+\sum_{i=1}^{n}\frac{\partial v_i}{\partial x_i}=0,\quad \frac{\partial u}{\partial x_i}=\frac{\partial v_i}{\partial y},\quad i=1,2,\cdots,n,$$

$$\frac{\partial v_i}{\partial x_j}=\frac{\partial v_j}{\partial x_i},\quad i\neq j,\quad 1\leqq i,j\leqq n,$$

and whose norm is defined by

$$||F||_p=\lim_{y\to 0}\left\{\int_{E_n}|F(X,y)|^p dx\right\}^{1/p}.$$

Let $f\in L_p(-\pi,\pi)(p\geqq 1)$ be periodic with period 2π, then its conjugate function is defined by

$$\tilde{f}(x)=\frac{1}{\pi}P.V.\int_{-\pi}^{\pi}\frac{f(y)}{2\tan(x-y)/2}dy.$$

One of its n-dimensional analogue is M. Riesz transform;

$$(Rf)(X)=((R_1f)(X),\cdots,(R_nf)(X))=\frac{1}{c_n}P.V.\int\frac{X-Y}{|X-Y|^{n+1}}f(Y)dY,$$

where $c_n=\pi^{(n+1)/2}/\Gamma((n+1)/2)$, and $f\in L_p(E_n)$.

We remark that if we put $Kf=(f+i\tilde{f})/2$ for $f\in L_p(-\pi,\pi)(p>1)$, then $Kf\in H_p$ and in particular if $f\in H_p(p\geqq 1)$, then $Kf=f$. Similarly if we put $\mathfrak{R}f=(f,Rf)=(f,R_1f,\cdots,R_nf)$ for $f\in L_p(E_n)(p>1)$, then f is a boundary function in $\mathfrak{H}_p$ and conversely if $F=(f,f_1,\cdots,f_n)$ is a boundary function in $\mathfrak{H}_p$, then $\mathfrak{R}f=F$.

§2. Let T be a quasi-linear operator from $\mathfrak{H}_p$(or H_p) to ν-

1) We denote the Euclidean space of n-dimension by E_n, its points $(x_1,\cdots,x_n)$, $(y_1,\cdots,y_n)$, etc. by X, Y, etc. and the element of volume $dx_1dx_2\cdots dx_n$ by dX.