159. On Distributions and Spaces of Sequences. IV

On Generalized Multiplication of Distributions

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1. Introduction. In the previous article [2] published under the same title, we considered the equivalent classes $\mathfrak{c}(T|\tau_1|\tilde{T}_\alpha)$, $\mathfrak{c}(T|\tau_1|\infty_\beta)$ and the ranges of product $\boldsymbol{R}[\mathfrak{c}(T|\tau_1|\ )\cdot\mathfrak{c}(S|\tau_2|\ )]_{\mathfrak{D}'}$, $\boldsymbol{R}[\mathfrak{c}(T|\tau_1|\ )\circ\mathfrak{c}(S|\tau_2|\ )]_{\mathfrak{D}'}$.

In this article we investigate detail relations between the topologies τ_1, τ_2 and the ranges $\boldsymbol{R}[\mathfrak{c}(T|\tau_1|\ )\cdot\mathfrak{c}(S|\tau_2|\ )]_{\mathfrak{D}'}$, $\boldsymbol{R}[\tau(T|\tau_1|\ )\circ\mathfrak{c}(S|\tau_2|\ )]_{\mathfrak{D}'}$. We also give here full explanation to our considerations which are discussed in [1] about Theorem given by L. Schwartz. We add here also some corrections to the errors found in the previous articles [I] and [2].

2. Notations and Definitions. We consider the set of all sequences $\{\varphi_n\}$ of functions $\varphi_n\in(\mathcal{E})$. In this set we introduce the following relations:

(1) $\{\psi_n\}=\{\varphi_n\}\Longleftrightarrow\psi_n=\varphi_n$ for all n,

(2) $\{\psi_n\}\pm\{\varphi_n\}=\{\psi_n\pm\varphi_n\}$,

and construct the linear space $\boldsymbol{Q}$.

Let $\tilde{\boldsymbol{Q}}_\tau$ denote the subspace of all convergent sequences in τ topology (on $(\mathcal{E})$), where τ is a topology which is finer than $\tau_{\mathfrak{D}'}$ and is compatible with the linear operations in $(\mathcal{E})$.

Let $\boldsymbol{O}_\tau$ denote the set of all sequences which converge to zero in τ topology. Let Q_τ denote the set of classes such that $Q_\tau\equiv\boldsymbol{Q}/O_\tau$ $=\{\mathfrak{c}(\ |\tau|\tilde{T}_\alpha),\ \mathfrak{c}(\ |\tau|\infty_\beta)\}$.

Let $\tilde{Q}_\tau$ be the set of all convergent classes, i.e.

$$\tilde{Q}_\tau\equiv\tilde{\boldsymbol{Q}}_\tau/\boldsymbol{O}_\tau=\{\mathfrak{c}(\ |\tau|\tilde{T}_\alpha)\}.$$

We consider the set of all convergent (in $\tau_{\mathfrak{D}'}$) sequences $\{\varphi_n\}$, $\varphi_n\in(\mathcal{E})$. In this set, we introduce the above relations (1), (2), and construct the linear space $\boldsymbol{Q}^{\mathfrak{D}'}$. Let $\tilde{\boldsymbol{Q}}^{\mathfrak{D}'}_\tau$ denote the subspace of all convergent sequences in τ topology which is contained in $\boldsymbol{Q}^{D'}$. Let $Q^{\mathfrak{D}'}_\tau$ be the set of all classes; $Q^{\mathfrak{D}'}_\tau\equiv\boldsymbol{Q}^{\mathfrak{D}'}/\boldsymbol{O}_\tau=\{\mathfrak{c}(T|\tau|\tilde{T}_\alpha),\ \mathfrak{c}(T|\tau|\infty_\beta)\}$, where $\varphi_n\in\mathfrak{c}(T|\tau|\tilde{T}_\alpha)$ means φ_n converge to T in $(\mathfrak{D}')$, and φ_n converge to $\tilde{T}_\alpha$ in τ. Let $\tilde{Q}^{\mathfrak{D}'}_\tau$ denote the set of all convergent classes of $Q^{D'}_\tau$ i.e. $\tilde{Q}^{\mathfrak{D}'}_\tau\equiv\tilde{\boldsymbol{Q}}^{\mathfrak{D}'}_\tau/\boldsymbol{O}_\tau$ $=\{\mathfrak{c}(T|\tau|\tilde{T}_\alpha)\}$.

Let P_τ be the natural mapping from $\boldsymbol{Q}$ to Q_τ or $\boldsymbol{Q}^{\mathfrak{D}'}$ to $Q^{\mathfrak{D}'}_\tau$.