158. On Distributions and Spaces of Sequences. III

On Powers of the Distributions

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1. In the previous articles [1] and [2] published under the same title, we considered the convergent or divergent generalized distributions. In this article we investigate the topologies defined by the powers of sequences and the derived generalized distributions introduced by these topologies. We also show here corrections to the errors in the previous articles [1], [2], and [3].

2. **Definition 1.** We denote σ_α the topology induced in the space $\mathfrak{D}$ by neighbourhoods $U_n(\varphi)$ of φ:

$U_n(\varphi)=\{\psi \mid e^{ia(\psi)}|\psi|^\alpha - e^{ia(\varphi)}|\varphi|^\alpha \in W_{\mathfrak{D}'}(0)\}$ where $a(\varphi), a(\psi)$ means the argument of the complex valued function φ or ψ and $\alpha>0$.

Then we see the following

Lemma 1. *The topology σ_α defines an uniform Hausdorff structure of the space $\mathfrak{D}$.*

Proof. It is easily seen that σ_α satisfies the conditions of Hausdorff space, and also the conditions $(\mathrm{U_I})$, $(\mathrm{U_{II}})$ of the uniformity [4]. Since for any $W_{\mathfrak{D}'}(0)$ there exists $W'_{\mathfrak{D}'}(0)$ such that $W'-W'\subset W$, the condition $(\mathrm{U_{III}})$ is satisfied.

Definition 2. We say that $\{\varphi_n\}$ is a *Cauchy sequence in σ_α*, if and only if the following condition is satisfied:

For any $W_{\mathfrak{D}'}(0)$, there exists an integer N such that $\varphi_n \in U_W(\varphi_n)$ for all $m, n>N$.

Definition 3. Suppose that two Cauchy sequences $\{\varphi_n\}$ and $\{\psi_n\}$ satisfy the following condition: for any $W_{\mathfrak{D}'}(0)$, there exists an integer N such that $\varphi_n \in U_W(\psi_m)$, $\psi_m \in U_W(\varphi_n)$ for all $m, n>N$. Then we say that $\{\varphi_n\}$ is *equivalent* to $\{\psi_m\}$.

Lemma 2. *The topology σ_α is not compatible with linear operation.*

Proof. We show here that there exist two sequences $\{\varphi_j\}$ and $\{\psi_j\}$ such that though both φ_j and ψ_j converge to 0, the sequence $\varphi_j+\psi_j$ does not converge to 0.

Example 1. φ_n is defined by $\varphi_n^0 * \rho_{n^2}$ where

$$\varphi_n^0=\begin{cases} n^{1/\alpha} & \text{for} \quad 1/n\le x\le 2/n \\ -n^{1/\alpha} & \text{for} \quad -2/n\le x\le -1/n \end{cases}$$

and ρ_{n^2} is a mollifier defined by L. Schwartz [5] with a compact