157. Some Applications of the Functional-Representations of Normal Operators in Hilbert Spaces. V

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In this paper, for the function $S(\lambda)$ defined in the statement of Theorem 1 we shall discuss its behaviour in the exterior of an appropriately large circle with center at the origin and shall show the orthogonality between its ordinary part and its two principal parts.

Theorem 13. Let $S(\lambda)$ and $\{\lambda_\nu\}$ be the same notations as those defined in the statement of Theorem 1, and let $M_S(\rho, 0)$ denote the maximum of the modulus $|S(\lambda)|$ as λ ranges over the points of the circle $|\lambda|=\rho$ satisfying $\sup_\nu|\lambda_\nu|<\rho<\infty$. Then a necessary and sufficient condition that the ordinary part $R(\lambda)$ of $S(\lambda)$ be a polynomial in λ of degree less than or equal to d is that there exist a positive number K and an appropriately large number σ such that the inequality

(12) $$M_S(r, 0)\geqq Kr^d$$

holds for every r satisfying $\sup_\nu|\lambda_\nu|<\sigma<r<\infty$.

Proof. If we put

$$\begin{cases} a_n=\dfrac{1}{\pi}\displaystyle\int_0^{2\pi}S(\rho e^{it})\cos nt\,dt \\ b_n=\dfrac{1}{\pi}\displaystyle\int_0^{2\pi}S(\rho e^{it})\sin nt\,dt \end{cases}\qquad (n=0, 1, 2, \cdots)$$

where ρ is an arbitrarily given number such that $\sup_\nu|\lambda_\nu|<\rho<\infty$, then, as shown in Theorem 7, for any κ subject to the condition $0<\kappa<1$ we have

$$S(\rho e^{i\theta}/\kappa)=\frac{a_0}{2}+\frac{1}{2}\sum_{n=1}^{\infty}(a_n-ib_n)(e^{i\theta}/\kappa)^n+\frac{1}{2}\sum_{n=1}^{\infty}(a_n+ib_n)(\kappa/e^{i\theta})^n$$

(θ: variable),

where the series on the right-hand side converges absolutely and uniformly, and hence

$$\frac{1}{2\pi}\int_0^{2\pi}|S(\rho e^{i\theta}/\kappa)|^2\,d\theta=\frac{1}{4}\Big(|a_0|^2+\sum_{n=1}^{\infty}|a_n-ib_n|^2\frac{1}{\kappa^{2n}}+\sum_{n=1}^{\infty}|a_n+ib_n|^2\kappa^{2n}\Big).$$

The final equality here implies that

$$\frac{1}{2}\Big(|a_0|^2+\sum_{n=1}^{\infty}|a_n-ib_n|^2\frac{1}{\kappa^{2n}}+\sum_{n=1}^{\infty}|a_n+ib_n|^2\kappa^{2n}\Big)^{\frac{1}{2}}\leqq M_S(\rho/\kappa, 0),$$

so that