156. On the Summability Methods of Logarithmic Type

By Kazuo ISHIGURO
Department of Mathematics, Hokkaido University, Sapporo
(Comm. by K. KUNUGI, M.J.A., Dec. 12, 1962)

§1. When a sequence $\{s_n\}$ is given we define the method l as follows: If

(1) $$t_0=s_0,\ t_1=s_1,$$
$$t_n=\frac{1}{\log n}\left(s_0+\frac{s_1}{2}+\cdots+\frac{s_n}{n+1}\right)\quad (n\geq 2)$$

tend to a finite limit s as $n\to\infty$, we say that $\{s_n\}$ is summable (l) to s and write $\lim s_n=s(l)$. (See [3], p. 59, p. 87.)

On the other hand we define the method L as follows: If

(2) $$\frac{-1}{\log(1-x)}\sum_{n=0}^{\infty}\frac{s_n}{n+1}x^{n+1}$$

tends to a finite limit s as $x\to 1$ in the open interval $(0,1)$, we say that $\{s_n\}$ is summable (L) to s and write $\lim s_n=s(L)$. (See [2].)

Concerning these methods we know the following theorems.

Theorem 1. *If $\{s_n\}$ is Cesàro summable $(C,1)$ to s, then it is summable (l) to the same sum. There is a sequence summable (l) but not summable $(C,1)$.* (See [3], p. 59, [5], p. 32.]

Theorem 2. *If $\{s_n\}$ is Abel summable (A) to s, then it is summable (L) to the same sum. There is a sequence summable (L) but not summable (A).* (See [2], [3], p. 81.)

Here we establish the following theorems.

Theorem 3. *If $\{s_n\}$ is summable (l) to s, then it is summable (L) to the same sum.*

Theorem 4. *If $\{s_n\}$ is summable (l) to s, then*
$$s_n=o(n\log n).$$
Furthermore if we put
$$s_n=a_0+a_1+\cdots+a_n\quad (n\geq 0),$$
we get
$$a_n=o(n\log n)$$
from the summability (l) of $\{s_n\}$.

Theorem 5. *There is a sequence summable (L) but not summable (l).*

§2. **Proof of Theorem 3.** From (1) we get
$$t_0=s_0,\ t_1=s_1,$$
$$t_n\log n=s_0+\frac{s_1}{2}+\cdots+\frac{s_n}{n+1}\quad (n\geq 2),$$

or