6. *On the Images of Connected Pieces of Covering Surfaces. I*

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Let $w=f(z)$ be an analytic function in $|z|<1$. It is interesting to consider the distribution of zero-points of $f(z)-a=0$. Suppose $f(z)$ is of bounded type. Let $\{a_i\}$ be the set of a-points of $f(z)$. Then it is well known that $\sum_i G(z, a_i)<\infty$, where $G(z, a_i)$ is the Green's function of $|z|<1$. In the present paper we consider the distribution of the set $f^{-1}(C(\rho, w_0))$ in $|z|<1$, where $C(\rho, w_0)=E[w: |w-w_0|<\rho]$.

Let R be a Riemann surface with positive boundary and let $R_n(n=0, 1, 2,\cdots)$ be its exhaustion with compact relative boundary ∂R_n. Let $G\subset G'$ be domains[1] in R, where G and G' may consist of at most enumerably infinite number of components. Let $w_n(z)$ be the least positive superharmonic function in G' such that $w_n(z)\geqq 1$ on $G\cap(R-R_n)$. Put $w(B\cap G, z, G')=\lim_n w_n(z)$ and call it[2] H.M. of $(G\cap B)$. If there exists a number n_0 such that $D(\omega_n(z))<M<\infty$ for $n\geqq n_0$, where $\omega_n(z)$ is a harmonic function in G' such that $\omega_n(z)=1$ on $G\cap R-R_n)$, $=0$ on $\partial G'$ and has M.D.I. (minimal Dirichlet integral), $\omega_n(z)\to$[2] in mean to $\omega(G\cap B, z, G')$ called C.P. of $(G\cap B)$. In case $G'=R$, we write $w(G\cap B, z)$ and if $G'=R-R_0$, we write $\omega(G\cap B, z)$ simply. Put $S(G, r)=E[z\in G: |z|=r]$.

Let G be a domain (of one component) in $|z|<1$. If there exists no bounded harmonic function in G vanishing on ∂G, i.e. $w(G\cap B, z, G)=0$, we say that G is *almost compact*. Let $C(\rho, w_0)$ be a circle in the w-plane. Then $f^{-1}(C(\rho, w_0))$ is composed of at most enumerably infinite number of components (connected pieces) $g_1, g_2,\cdots$. If a domain G is a subset of $\{g_i\}$, we call G a D.G. (domain generated) of $f^{-1}(C(\rho,w_0))$. At first we shall prove by simple method the following

Theorem 1. *Let $w=f(z)$ be an analytic function in $|z|<1$ such that $|f(z)|\leqq M$.*

a) *Let G be a* D.G. *of $f^{-1}(C(\rho, w_0))$ and let G' be a* D.G. *of $f^{-1}(C(\rho' w_0))$ containing $G:\rho<\rho'$. Then $w(G\cap B, z)>0$ if and only if there exists at least one component g' of G' such that $w(G\cap B, z, g')>0$ for any $\rho'>\rho$.*

1) In the present paper we suppose the relative boundary of a domain consists of analytic curves clustering nowhere in R.

2) Z. Kuramochi: Potentials on Riemann surfaces: Journ. Sci. Hokkaido Univ. **14** (1962).