5. On a Boundary Theorem on Open Riemann Surfaces

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1. **Introduction.** Let U be the class of Riemann surfaces on which there exist the Green function and at least a bounded minimal positive harmonic function (C. Constantinescu and A. Cornea [1]) and O_L be the class of Riemann surfaces on which there exist the Green function and no non-constant Lindelöfian meromorphic function (M. Heins [3]). Let R be an open Riemann surface and Ω be a subregion of the Riemann surface R whose relative boundary $\partial\Omega$ with respect to R consists of at most an enumerable number of analytic curves clustering nowhere in R. If there exists no non-constant single-valued bounded harmonic function in Ω which vanishes continuously on $\partial\Omega$, we say that Ω belongs to SO_{HB}. The following theorem was proved by many authors (see [2], [4], [6], and [8]).

Let R be an open Riemann surface belonging to the class U and Ω be a subregion of R which satisfies the above boundary condition and does not belong to SO_{HB}, then Ω belongs to O_L.

In the present paper we shall give another simple proof of this assertion with aid of the notion of thinness in Martin's space [5] (which is given by Martin's compactification of an open Riemann surface), introduced by L. Naïm [7].

2. **Preliminaries.** We shall introduce the notion of thinness and some useful results for our purpose.

Let R be an open Riemann surface and $\hat{R}$ be Martin's space associated with R. We say that $\Delta^R=\hat{R}-R$ is the Martin boundary of R. Now let $K_x(y)$ be a kernel function in the sense of Martin, that is $K_x(y)=\dfrac{G(x, y)}{G(x, y_0)}$ for $x\in\hat{R}-\{y_0\}$, $y\in R$ with a fixed point y_0 in R. Then x_0 is said to be a minimal point of Δ^R if $K_{x_0}(y)$ is a minimal positive harmonic function in R in the sense of Martin and x_0 is said to be a bounded minimal point of Δ^R if, in addition, $K_{x_0}(y)$ is bounded in R.

Let m be a positive measure in R, then a K-potential with respect to the measure m in R is defined in $\hat{R}-\{y_0\}$ by

$$U(x)=\int K_x(y)\,dm(y).$$

Definition. *A subset E of R is said to be thin at a point x_0 in*