3. On the Existence and the Propagation of Regularity of the Solutions for Partial Differential Equations. I

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1. Introduction. The object of this note is to derive a priori inequality based on our recent note [4], which is applicable to the existence theorem and the propagation of regularity of the solutions for partial differential equations.

Recently L. Hörmander [2] has already derived a similar inequality under some conditions for the principal part of given operators.

We shall consider differential operator L in a neighborhood of the origin in $(\nu+1)$-space: $(t, x)=(t, x_1, \cdots, x_\nu)$. Let $(m, \mathfrak{m})=(m, m_1, \cdots, m_\nu)(m_j \leqq m; j=1, \cdots, \nu)$ be an appropriate real vector whose elements are positive integers. The operator considered in this note is of the form

$$(1.1) \qquad L=L_0+\sum_{i+m|\alpha:\mathfrak{m}|\leqq m-1} b_{i,\alpha}(t, x)\frac{\partial^{i+|\alpha|}}{\partial t^i \partial x^\alpha}$$

with

$$(1.2) \qquad L_0=\sum_{i+m|\alpha:\mathfrak{m}|=m} a_{i,\alpha}(t, x)\frac{\partial^{i+|\alpha|}}{\partial t^i \partial x^\alpha} \quad (a_{m,0}(t, x)=1)$$

$$(\alpha=(\alpha_1, \cdots, \alpha_\nu),\ x^\alpha=x_1^{\alpha_1}\cdots x_\nu^{\alpha_\nu},\ |\alpha|=\alpha_1+\cdots+\alpha_\nu,\ |\alpha:\mathfrak{m}|=\alpha_1/m_1+\cdots+\alpha_\nu/m_\nu)$$

where $b_{i,\alpha}$ are in L^∞ and $a_{i,\alpha}$ in $C^\infty_{(t,x)}$.[1]

Setting for (1.2) and real vectors $\xi=(\xi_1, \cdots, \xi_\nu)$

$$(1.3) \qquad L_0(t, x, \lambda, \xi)=\sum_{i+m|\alpha:\mathfrak{m}|=m} a_{i,\alpha}(t, x)\lambda^i \xi^\alpha$$

which we call the characteristic polynomial of L, we derive a priori inequality (3.3) under some conditions for the characteristic roots $\lambda=\lambda(\xi)$ of the equation $L_0(t, x, \lambda, \sqrt{-1}\,\xi)=0$ for $\xi \neq 0$.

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2. Definitions and lemmas. Let us define $r=r(\xi)$ for real vector ξ as a positive root of the equation

$$(2.1) \qquad F(r, \xi)\equiv \sum_{j=1}^{\nu} \xi_j^2 r^{-2/m_j}=1 \quad (\xi \neq 0).$$

Then, r is in $C^\infty_{(\xi\neq 0)}$ and satisfies inequalities

1) Strictly speaking it is sufficient to assume that $a_{i,\alpha}$ are in $C^k_{(t,x)}$ for $k \geqq m+(\nu+1)\underset{1\leqq j\leqq \nu}{Max}\, m/m_j$.