24. Inversive Semigroups. II

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This paper is the continuation of the previous paper Yamada [1]. Any terminology without definition should be referred to [1]. In this paper, we shall present necessary and sufficient conditions for an inversive semigroup to be isomorphic to some special subdirect product of a group and a band.

The proofs of any theorems and corollaries are omitted and will be given in detail elsewhere.[1)]

§1. **Group-semilattices.** Let G be a group, and Γ a semilattice. Let $\{G_\gamma:\gamma\in\Gamma\}$ be a collection of subgroups G_γ of G such that

(1) $\bigcup\{G_\gamma:\gamma\in\Gamma\}=G$

and (2) if $\alpha\leqq\beta$ (i.e., $\alpha\beta=\beta\alpha=\alpha$) then $G_\alpha\supset G_\beta$.

Let $S=\sum_{\gamma\in\Gamma}G_\gamma$, where $\sum$ denotes the class sum (i.e., the disjoint sum) of sets. If $x\in G$ is an element of G_γ, then we denote x by (x,γ) when we regard x as an element of G_γ in S. Now, S becomes a semigroup under the multiplication $\circ$ defined by the following

(P) $(x,\alpha)\circ(y,\beta)=(xy,\alpha\beta)$.

That is, S is a compound semigroup of $\{G_\gamma:\gamma\in\Gamma\}$ by Γ,[2)] and accordingly a (C)-inversive semigroup. We shall call such an S a *group-semilattice* of G, and denote by $\{G_\gamma|\Gamma,G\}$. Moreover, in this case we shall call G the *basic group* of S. Now, let I be a band whose structure decomposition is $I\sim\sum\{I_\gamma:\gamma\in\Gamma\}$.[3)] Then, we can consider the spined product of S and I with respect to Γ, because S and I have the same structure semilattice Γ.

As a connection between subdirect products of G and I and the spined product of S and I, we have

Theorem 1. *The spined product of group-semilattice of G and a band I is isomorphic to an inversive subdirect product of G and I.*[4)] *Conversely, any inversive subdirect product of a group G and a band I is isomorphic to the spined product of a group-semilattice of G and I.*

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1) This is an abstract of the paper which will appear elsewhere.

2) For compound semigroups, see M. Yamada, Compositions of semigroups, Kōdai Math. Sem. Rep., **8**, 107–111 (1956).

3) For the definition of the structure decomposition of a band, see N. Kimura, Note on idempotent semigroups. I, Proc. Japan Acad., **33**, 642–645 (1954).

4) Let D be a subdirect product of G and I. Then, D is clearly a semigroup. If D is an inversive semigroup, then D is called an *inversive subdirect product* of G and I.