21. *Normality and Perfect Mappings*

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We assume that the spaces considered here are always completely regular T_1-spaces. A mapping φ from X onto Y is said to *be perfect* if φ is a closed continuous mapping and every $\varphi^{-1}(y)$, $y\in Y$, is compact, i.e., φ is a compact mapping. Let E be any dense subspace of a given space X. It is easy to see that the normality of $X\times\beta E$ implies the normality of $X\times BE$ where BE is any compactification of E and βE is the Stone-Čech compactification of E. But the following problem is open [1, § 4].

(*) *Does the normality of $X\times BE$ implies the normality of $X\times\beta E$?*

This problem is closely related to the following open problem [1, problem 4]:

(**)[1] *Let φ be a perfect mapping from X onto Y such that the image of any proper closed subset of X is a proper closed subset of Y. Is it true that X is normal whenever Y is normal?*

In § 1, we shall investigate some special class of spaces, and, in § 2, we shall give the negative answers to the problems (*) and (**).

In the sequel, ω_α denotes the smallest ordinal of cardinal $\aleph_\alpha$ and we mean by $W(\omega_\alpha)$ the set of all cardinals less than ω_α; then $W(\omega_\alpha)$ $(\alpha\neq 0)$, endowed the interval topology, is a countably compact normal space and there are no subsets of cardinal $<\aleph_\alpha$ which are cofinal [6, 9K].

1. Closedness of projections. We mean by φ(or φ_X): $X\times Y\to X$ the projection $\varphi(x,y)=x$ from $X\times Y$ onto X. Let $\mathfrak{N}$ be the class consisting of all X such that $\varphi: X\times Y\to X$ is always closed for any countably compact space Y.

1.1. Lemma. *If X has the property such that for any point p and any subset E of X, there is a sequence in E converging to p whenever p is an accumulation point of E, then X belongs to $\mathfrak{N}$.*

Proof. Let Y be a countably compact space and F a closed subest of $X\times Y$ such that the image E of F under $\varphi: X\times Y\to X$ is not closed. There is a point p in $\overline{E}-E$. By the assumption, there is a sequence (x_n) in E converging to p. Let (x_n, y_n) be a point of F for every n. Since Y is countably compact, there is an accumula-

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1) This problem is raised by Nagami [7] in connection with Ponomarev's theorem [8].