20. On the Composition of a Summable Function and a Bounded Function

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1. Introduction. The main purpose of this paper is to argue the generalized harmonic analysis of a function of composition type in the Weyl space. Let $f(x)$ be a bounded measurable function and $K(x)$ be a summable function on $(-\infty, \infty)$. Let us consider the composition of f and K:

$$g(x)=\int_{-\infty}^{\infty} K(x-y)f(y)dy=K*f. \tag{1.1}$$

Let us denote by $s(u, x)$ the Fourier-Wiener transform of $f(x+t)$ where we take "t" as variable:

$$\begin{aligned} s(u, x)= &\ \underset{A\to\infty}{\text{l.i.m.}}\frac{1}{\sqrt{2\pi}}\left[\int_1^A+\int_{-A}^{-1}\right]\frac{f(x+t)e^{-iut}}{-it}dt \\ &+\frac{1}{\sqrt{2\pi}}\int_{-1}^{1} f(x+t)\frac{e^{-iut}-1}{-it}dt. \end{aligned} \tag{1.2}$$

Let us introduce the norm which was firstly defined by H. Weyl [1] in the study of almost periodic functions. It concerns with measurable and integrable function in any finite interval and such that

$$\overline{\lim_{l\to\infty}}\sup_{-\infty<x<\infty}\frac{1}{l}\int_x^{x+l}|f(t)|^2\,dt<\infty. \tag{1.3}$$

By $f\sim g$ we mean that we have

$$\overline{\lim_{l\to\infty}}\sup_{-\infty<x<\infty}\frac{1}{l}\int_x^{x+l}|f(t)-g(t)|^2\,dt=0. \tag{1.4}$$

For the sake of simplicity we use the notation

$$||f||_p=\left(\int_{-\infty}^{\infty}|f(t)|^p\,dt\right)^{1/p} \quad (p>0). \tag{1.5}$$

Then the main result of this paper is as follows:

Theorem 1. Let $f(x)$ and $g(x)$ be bounded measurable functions on $(-\infty, \infty)$. Let $K(x)$ be a measurable function of the class $L_1(-\infty, \infty)$. Let us denote by $s(u, x)$ and $t(u, x)$ the Fourier-Wiener transform of $f(x+t)$ and $g(x+t)$ respectively. Let us put

$$I(\varepsilon, x)=\frac{1}{\varepsilon}||\{t(u+\varepsilon, x)-t(u-\varepsilon, x)\}-k(u)\{s(u+\varepsilon, x)-s(u-\varepsilon, x)\}||_2^2 \tag{1.6}$$

where $k(u)$ is the Fourier transform of $K(t)$. Then under the supplementary condition