17. *Evans' Harmonic Functions on Riemann Surfaces*

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1. Besides its own interest, Evans' harmonic function on open Riemann surfaces is important in the function theory on open Riemann surfaces. In this note, we shall sketch a method to construct Evans' harmonic function on open Riemann surfaces. The detail will be published elsewhere.

DEFINITION (*Boboc-Constantinescu-Cornea* [1]). *Let R be a hyperbolic Riemann surface and $\mathfrak{F}$ be the class of all sequences $(z_n)_{n\geq 1}$ of points in R which do not accumulate in R and*

$$\liminf_{n\to\infty} g(z_n, z_0) > 0,$$

where $g(z, z_0)$ is Green's function on R with its pole z_0 in R. An Evan's function $S(z)$ on R is a positive continuous superharmonic function on R such that

$$\lim_{n\to\infty} S(z_n) = \infty$$

for any (z_n) in $\mathfrak{F}$. Moreover if $S(z)$ is harmonic on R, we call $S(z)$ an Evans' harmonic function on R.

Boboc, Constantinescu and Cornea [1] proved the existence of Evans' function on R. In the case where $R=R'-\overline{R}'_0$, where R' is a parabolic Riemann surface and R'_0 is a relatively compact subdomain of R' with smooth boundary, Kuramochi [2] proved the existence of Evans' harmonic function on R, from which the existence of Evans-Selberg's potential on R' follows at once by using the linear operator method of Sario [8]. The present author [6] gave an alternating proof of Kuramochi's result. Here we state the following

THEOREM. *There exists an Evan's harmonic function on hyperbolic Riemann surfaces.*

2. For the proof of our theorem, we use the theory be Royden's compactification. The present method to construct the desired function is already used partly in [6] and [7].

Let R be an arbitrary Riemann surface and $M(R)$ be the *Royden's algebra* associated with R, i.e. the algebra of all complex-valued absolutely continuous functions in the sense of Tonelli which are bounded and of finite Dirichlet integral. The algebra $M(R)$ is a Banach algebra with the norm $||f||=\sup(|f(z)|; z\in R)+\sqrt{D(f)}$ and the subalgebra $M(R)\cap C^\infty(R)$ is dense in $M(R)$ with respect to this norm. Hence Green's formula and the Dirichlet principle can be freely applied to functions in $M(R)$ ([3]).