# 35. On the Product of a Normal Space with a Metric Space

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Let $X$ be a topological space. Then the topological product of $X$ with every metrizable space is proved to be normal for the following three cases.

I. $X$ is paracompact and perfectly normal (E. Michael [2]).

II. $X$ is paracompact and topologically complete in the sense of E. Čech (Z. Frolik [1]).

III. $X$ is countably compact and normal (A. H. Stone [4]).

Quite recently E. Michael [3] has shown that the product space $X\times Y$ is not normal in general even if $X$ is a hereditarily paracompact Hausdorff space with the Lindelöf property and $Y$ is a separable metric space.

In view of these facts it is desirable to find a necessary and sufficient condition for $X$ to possess the property that the product space $X\times Y$ be normal for any metrizable space $Y$. This problem, however, was open until now (cf. H. Tamano [5]). The purpose of this note is to give a solution to this problem. The proofs and the details of the results will be published elsewhere.

1. Let us consider the following condition for a topological space $X$.

For any set $\Omega$ of indices and for any family $\{G(\alpha_1,\cdots,\alpha_i)\mid \alpha_1,\cdots,\alpha_i\in\Omega;\ i=1,2,\cdots\}$ of *open* subsets of $X$ satisfying the condition

(1) $$G(\alpha_1,\cdots,\alpha_i)\subset G(\alpha_1,\cdots,\alpha_i,\alpha_{i+1}) \quad \text{for} \quad \alpha_1,\cdots,\alpha_{i+1}\in\Omega \ \text{and for} \ i=1,2,\cdots$$

there exists a family $\{F(\alpha_1,\cdots,\alpha_i)\mid \alpha_1,\cdots,\alpha_i\in\Omega,\ i=1,2,\cdots\}$ of *closed* subsets of $X$ satisfying the following two conditions:

(2) $$F(\alpha_1,\cdots,\alpha_i)\subset G(\alpha_1,\cdots,\alpha_i) \quad \text{for} \quad \alpha_1,\cdots,\alpha_i\in\Omega.$$

(3) $$\text{If} \quad \bigcup_{i=1}^{\infty} G(\alpha_1,\cdots,\alpha_i)=X, \ \text{then} \ \bigcup_{i=1}^{\infty} F(\alpha_1,\cdots,\alpha_i)=X.$$

We shall say that $X$ is a *P-space* if $X$ satisfies the above condition.

As is well known, a normal space $X$ is countably paracompact if and only if for any countable open covering $\{G_i\}$ of $X$ with $G_i\subset G_{i+1}$, $i=1,2,\cdots$ there exists a countable closed covering $\{F_i\}$ of $X$ such that $F_i\subset G_i$, $i=1,2,\cdots$. Hence a normal $P$-space is always countably paracompact. On the other hand, it follows from an example of Michael [3], in view of our Theorem 2.1 below, that a