51. *Remarks on Some Properties of Solutions of Some Boundary Value Problems for Quasi-linear Parabolic and Elliptic Equations of the Second Order*

By Takaŝi KUSANO

Chuo University, Tokyo

(Comm. by Kinjirô KUNUGI, M.J.A., April 12, 1963)

Introduction. In this note we shall try to generalize some of the results established by Oleinik [6] and Výborný [7] for linear elliptic and parabolic differential equations of the second order. Namely, we shall consider second order quasi-linear parabolic and elliptic equations and discuss first the behavior of their solutions at the boundary of the domain where they attain positive maximum or negative minimum. Next we shall formulate the uniqueness theorems for some boundary value problems with oblique derivatives. In our discussion extensive use is made of the maximum principles proved by the author [8], [9] for quasi-linear elliptic and parabolic equations. Since the treatment is similar for both parabolic and elliptic cases, we shall limit ourselves in our exposition to the detailed consideration of parabolic equations, while for elliptic equations only the corresponding theorems will be stated.

The author has to acknowledge gratefully the encouragement and the criticism presented by Professor Masuo Hukuhara and Professor Yoshikazu Hirasawa during the preparation of this note.

§ 1. Quasi-linear parabolic equations. In this section we are concerned with quasi-linear parabolic equations of the form

$$(1)\qquad \sum_{i,j=1}^{n} a_{ij}(x, t, u, \text{grad } u)\frac{\partial^2 u}{\partial x_i \partial x_j} - \frac{\partial u}{\partial t} = f(x, t, u, \text{grad } u),$$

$$x=(x_1,\cdots,x_n),\ \text{grad } u=(\partial u/\partial x_1,\cdots,\partial u/\partial x_n).$$

We denote by D a bounded domain in the $(n+1)$-dimensional (x, t)-space bounded by two hyperplanes $t=0$ and $t=T>0$, and by a lateral surface S lying between these hyperplanes. The union of the surface S and the lower basis $B=\overline{D}\frown\{t=0\}$ is referred to as the normal boundary of D and is denoted by ∂D. We assume that the functions $a_{ij}(x, t, u, p)$ and $f(x, t, u, p)$ are defined in the domain $\mathfrak{D}$: $\{(x, t)\in D, |u|<\infty, \|p\|<\infty\}$ and are bounded in compact subset of $\mathfrak{D}$. We impose the following assumption on the lateral surface S of D: for each point $P(x, t)\in S$ there exists an $(n+1)$-dimensional sphere K_P including P on its boundary such that all the points of K_P lying in the strip $0<t\leqq T$ belong to $\overline{D}-\partial D$. Finally we assign to each point of S a direction l which makes an acute angle with the inwardly