46. A Remark on General Imbedding Theorems in Dimension Theory

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Once we have constructed [1] a universal n-dimensional set for *general* metric spaces which is a rather complicated subset of C. H. Dowker's generalized Hilbert space. In this brief note we shall show that we can find a simpler universal n-dimensional set in a countable product of H. J. Kowalsky's star-spaces.

On the other hand, we have found [2] in the product of a generalized Baire 0-dimensional space and the Hilbert-cube a universal countable-dimensional set for metric spaces with a σ-star-finite basis. A universal countable-dimensional set for *general* metric space will also be found in such a product of star spaces.

Let E_α, $\alpha\in A$ be a system of unit segments $[0, 1]$. By identifying all zeros in $\bigcup\{E_\alpha \mid \alpha\in A\}$ we get a set S. We introduce a metric in S as follows.

$$\begin{aligned}\rho(x, y) &= |x-y| \quad \text{if } x, y \text{ belong to the same segment,}\\ &= |x|+|y| \quad \text{if } x, y \text{ belong to the distinct segments.}\end{aligned}$$

Then we get a metric space S called the *star-space* with the index set A. H. J. Kowalsky [3] proved that

a topological space R is metrizable if and only if it can be imbedded in a countable product of star-spaces.

Now we can assert the following theorem in dimension theory.

Theorem 1. *A metric space R has (covering) dimension $\leqq n$ if and only if it can be imbedded in the subset K_n of a countable product P of star-spaces, where we denote by K_n the set of points in P at most n of whose non-vanishing coordinates are rational.*

Proof. To see $\dim K_n \leqq n$ we decompose K_n as $K_n=\bigcup_{i=0}^{n} K_i'$ for the sets K_i' of points in P just i of whose non-vanishing coordinates are rational. We consider a given class $a_j, j=1\cdots i$ of i rational numbers with $0<a_j\leqq 1$. Then the set of points in K_i' whose j-th coordinates are equal to a_j is a 0-dimensional closed subset of K_i'. This assertion is proved by the product theorem in dimension theory since it is easily seen that the set of irrational points and zero in a star-space has dimension 0. Hence it follows from the sum-theorem that K_i' as a countable sum of such closed sets is 0-dimensional. This implies by the decomposition theorem that $\dim K_n \leqq n$.