45. Continuity of Path Functions of Strictly Stationary Linear Processes

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Let $X(t)$, $-\infty<t<+\infty$, be a mean continuous purely non-deterministic weakly stationary process with $EX(t)=0$. Then, by Karhunen [5], $X(t)$ can be expressed in the following form.

$$(1) \qquad X(t)=\int_{-\infty}^{t} g(t-u)\,dZ(u),$$

where the function g is in $L_2(R)$ and dZ is an orthogonal random measure such that $E(dZ(u))^2=du$. Further, let $\mathfrak{M}_t(X)$, $\mathfrak{M}(X)$ and $\mathfrak{M}_t(Z)$ be closed linear manifolds spanned by $\{X(\tau);\ \tau\leq t\}$, $\{X(\tau);\ -\infty<\tau<+\infty\}$ and $\{Z(\tau)-Z(\tau');\ \tau,\tau'\leq t\}$, respectively. We can take g and dZ to satisfy $\mathfrak{M}_t(X)=\mathfrak{M}_t(Z)$, uniquely up to the constant multiple with absolute value one.

Next, following P. Lévy and Hida-Ikeda [2], we call $X(t)$ a *linear process* if $\mathfrak{M}_t(X)$ and $\mathfrak{M}_t^{\perp}(X)=\{$the orthogonal complement of $\mathfrak{M}_t(X)$ in $\mathfrak{M}(X)\}$ are mutually independent for each t.

PROPOSITION. *Let $X(t)$ be a strictly stationary process with canonical representation of the form* (1). *Then $X(t)$ is a linear process if and only if $Z_a(t)=Z(t)-Z(a)$, $t\geq a$, is a temporally homogeneous additive process for each a.*

The proof of 'if' part is found in Hida-Ikeda [2]. 'Only if' part is easily proved by the definition of canonical representation.

In the following we assume $X(t)$ to be strictly stationary and linear. We want to investigate properties of its path functions.

An additive process which is continuous in probability may be considered as a Lévy process by taking an appropriate version. Hence, by Lévy-Itô's decomposition, we can write

$$(2) \qquad Z(t)-Z(a)=\sqrt{v}(B_0(t)-B_0(a))+P(t)-P(a),$$

where $B_0(t)$ is the standard Brownian motion and $P(t)-P(a)$ is the Poisson part. Then (1) and (2) imply

$$(3) \qquad X(t)=\sqrt{v}\int_{-\infty}^{t} g(t-u)\,dB_0(u)+\int_{-\infty}^{t} g(t-u)\,dP(u).$$

We denote the first term on the right side by $X_1(t)$ and the second by $X_2(t)$. $X_1(t)$ is a Gaussian stationary process and the properties of its path functions are investigated by Hunt [3] and Belayev [1]. So we shall treat $X_2(t)$ and give a sufficient condition for the continuity