62. A Note on the Completion Theory

By Suketaka MITANI

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In this paper, a generalization of the theory of completion due to Dr. Cohen ([3] [4] see also [6]) is attempted.

It is well known that the constructive method of Cantor-Hausdorff for the completion of metric spaces can be generalized to the case of uniform spaces (see for example [2] and [9]).

In recent years, G. Areskin gave a theory free from the uniformity, namely, which is valid at least for the general regular spaces [1], and P. Holm treated the case when the space has a system of finite coverings. In this paper, we shall consider the case when this system of covering is not necessarily finite. We have already given the notion of the Cauchy filter for the spaces with arbitrary coverings [9]. Moreover, it can be shown by some easy examples that our method is applicable naturally to non-regular spaces.

An *extention* X' of topological space X, is such a topological space that $X \subseteq X'$ and $\overline{X} = X'$. A completion X^* of a topological space is such an extention that; there exists a completion of a topological space X having some quasi-topological property P and if X consists of all the rational numbers and its some property P then X^* is the set of all real number; X^* is uniquely determined, for X and its some property P; and etc.$\cdots$. The completion theory is the theory of these quasi-topological property P.

At first we must define the completion, when we deal with considerably general property P.

In uniform space, even in Dr. Cohen's case, P is a global property for a covering system, but in this paper, Theorem 1 shows P is local one for filters which dosen't converge any points.

A *filter base* $\mathfrak{f}$ in a set X is a family of non-void subsets of X such that, for any $A, B \in \mathfrak{f}$, $C \subseteq A \frown B$ for some $C \in \mathfrak{f}$.

A *filter* $\mathfrak{f}$ is a set X is a filter base such that, if $A \in \mathfrak{f}$ and $A \subseteq B$ then $B \in \mathfrak{f}$. Let $\mathfrak{f}$ be a filter base, then $\{A \mid A \supseteq C, C \in \mathfrak{f}\}$ is a filter, and it is said to be *generated* by $\mathfrak{f}$. When a set X is contained in another set X^*, any filter $\mathfrak{f}$ in X generates a filter $\mathfrak{f}^*$ in X^*.

In a topological space X, a filter base $\mathfrak{f}$ is said to be *convergent* to a point x of X if the neighborhood system of x is contained in the filter which is generated by $\mathfrak{f}$.

A bow $\mathfrak{B}$ of X is a base of open sets of X which has a certain index set Λ, and