61. A Remark on Gentzen's Paper "Beweisbarkeit und Unbeweisbarkeit von Anfangsfällen der transfiniten Induktion in der reinen Zahlentheorie". II

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(Comm. by Zyoiti SUETUNA, M.J.A., May 11, 1963)

In this paper we shall define systems $\mathfrak{S}_2$ and $\mathfrak{S}_3$ and prove the theorem stated in the first paper of this title for these systems.

Definition of the system $\mathfrak{S}_2$. $\mathfrak{S}_2$ is a system obtained from G^1LC modifying it as follows (cf. [8]):

1. Every beginning sequence of $\mathfrak{S}_2$ is of the form $D\rightarrow D$ or of the form $a=b$, $A(a)\rightarrow A(b)$ or a 'mathematische Grundsequenz' in the sense of the first paper.

2. The inference-schema 'induction' is added.

3. The inference V left on an f-variable of the form

$$\frac{F(V),\ \Gamma\rightarrow\Delta}{V\varphi F(\varphi),\ \Gamma\rightarrow\Delta}$$

is restricted by the condition that $V\varphi F(\varphi)$ is f-closed, i.e. $V\varphi F(\varphi)$ does not contain any free f-variable.

The proof of the theorem and the result (†) for $\mathfrak{S}_2$ can be performed in the same way as for $\mathfrak{S}_1$.

The definition of $\mathfrak{S}_3$. Let $I(a)$ and $a<^*b$ be two primitive recursive predicates. Let us assume that the following condition is satisfied: $<^*$ is a well-ordering of I, where I is $\{a\,|\,I(a)\}$.

Now the formal system $\mathfrak{S}_3$ is obtained as follows from G^1LC.

1. Every beginning sequence is of the form $D\rightarrow D$ or of the form $a=b$, $A(a)\rightarrow A(b)$ or the 'mathematische Grundsequenz' in the sense of the first paper or the following form.

$$I(a),\ A_j(a,b)\rightarrow G_j(a,b\{x,y\}(A_j(x,y)\wedge x<^*a))$$

(*)

$$I(a),\ G_j(a,b,\{x,y\}(A_j(x,y)\wedge x<^*a))\rightarrow A_j(a,b)\qquad j=0,1,2,\cdots.$$

Here $\{x,y\}$ is used instead of usual notations $\hat{x}\hat{y}$, λxy and $A_1, A_2, A_3,\cdots$ are new symbols for predicates. Moreover, $G_j (j=0,1,2,\cdots)$ are arbitrary formulas satisfying the following conditions:

a) $G_j(a,b,\alpha)$ does not contain $A_j, A_{j+1}, A_{j+2},\cdots$.

b) If $G_j(a,b,\alpha)$ contains the figures of the form $V\varphi A(\varphi)$, then $A(\beta)$ does not contain any bound f-variable.

2. The inference-schema called 'induction' is added.

3. The inference V left on an f-variaqle of the form