77. *Some Applications of the Functional-Representations of Normal Operators in Hilbert Spaces. VII*

By Sakuji INOUE
Faculty of Education, Kumamoto University
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On the assumption that the ordinary part $R(\lambda)$ of the function $S(\lambda)$ defined in the statement of Theorem 1 is a transcendental integral function, in the preceding paper [1] we have discussed, under some conditions, the relation between the distribution of ζ-points of $R(\lambda)$ and that of ζ-points of $S(\lambda)$ in the exterior of an appropriately large circle with center at the origin and have proved that, if each of $R(\lambda)$ and $S(\lambda)$ has its finite exceptional value for the exterior of that circle, the two exceptional values are identical under the same conditions as above. In the present paper, however, we shall discuss these problems without using those conditions from a different point of view.

Theorem 19. Let $S(\lambda)$, $R(\lambda)$, and $\{\lambda_\nu\}$ be the same notations as those defined in the statement of Theorem 1; let σ be a given constant satisfying the condition $\sup_\nu |\lambda_\nu| < \sigma < \infty$; let

$$C=\sup_n \left\{ \frac{1}{2\pi} \left| \int_0^{2\pi} S(\rho e^{it}) e^{int} dt \right| \right\} \quad (<\infty) \quad (\sup_\nu |\lambda_\nu| < \rho < \sigma < \infty);$$

let us suppose that in the exterior of the circle $|\lambda| = \sigma$ there exists an infinite subsequence $\{z_n\}_{n=1,2,3,\cdots}$ of mutually distinct ζ-points of $R(\lambda)$ such that

$$\left.\begin{array}{l} R(z_n)=\zeta \\ \sigma < |z_n| \leqq |z_{n+1}| \end{array}\right\} (n=1,2,3,\cdots), \quad |z_n| \to \infty \, (n\to\infty), \quad |z_{n+1}|-|z_n| \to 0 \quad (n\to\infty),$$

and that, for two systems of appropriately chosen positive numbers r_n and ε_n with $\varepsilon_n < C$, $n=1,2,3,\cdots$, satisfying the conditions $r_n \to 0$ $(n\to\infty)$ and $\inf_n \varepsilon_n \equiv \varepsilon > 0$ respectively, $|R(z_n + r_n e^{i\theta}) - \zeta|$ is greater than or equal to ε_n, irrespective of the value of $\theta \in [0, 2\pi]$; let L be the least positive integer of n subject to the condition

$$|z_n| > \frac{2C\rho}{\varepsilon} + r \quad (r = \max_n r_n);$$

and let Γ_p be the circle $|\lambda - z_{L+p}| = r_{L+p}$ for each value of $p=0,1,2,\cdots$. Then, in the interior of each Γ_p, $S(\lambda)$ and $R(\lambda)$ have the same number (counted according to multiplicity) of ζ-points; and moreover, if the above hypotheses on the sets $\{z_n\}$, $\{r_n\}$, and $\{\varepsilon_n\}$ are satisfied for every $\zeta (\neq \infty)$, not exceptional value of $R(\lambda)$, and if ξ is the finite excep-