

### 105. On the Completion of Algebraic Systems that Satisfy the Conditions $T_0$ and $T_3$

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(Comm. by Kinjirô KUNUGI, M.J.A., Sept. 12, 1963)

Continuing the study of the author's paper [3], we shall give in this paper a generalization of Prof. Nakayama's theorem of uniform algebraic system.

Our generalization is naturally got from the theorems of the paper [3] and the followings.

**PROPOSITION.** *A completion  $(X^*; \mathfrak{B}^*)$  of a  $T_2$  bow space is a  $T_2$  space if and only if for any leg  $\dagger$  in  $X$  and any point  $x \in X$ , there exist some body  $V$  of  $\dagger$ , and some neighborhood  $W$  of  $x$ , such that  $V \cap W = \emptyset$ . (This proposition is an ameliorated one of the proposition mentioned at the end of [3] and can be proved easily.)*

A  $T_3$  algebraic system  $G$  is a  $T_3$  space  $G$  which is an algebraic system such that for any composition  $\pi$  defined in  $G$ , the function  $f(a, b) = a\pi b$  ( $a, b \in G$ ) is a continuous one from  $G \times G$  into  $G$ , and that any mappings defined on it as algebraic system, are also continuous. We consider a  $T_3$  algebraic system  $G$ , specially, with a bow  $\mathfrak{B}$ , such that  $(G; \mathfrak{B}, A)$  is a bow space, and let's call it a *bow algebraic system*. Further, if a bow algebraic system is complete as a bow space, then the bow algebraic system is said to be *complete*.

A *completion*  $(G^*; \mathfrak{B}^*)$  of a bow algebraic system  $(G; \mathfrak{B})$  is a bow algebraic system such that;

(G<sub>1</sub>) the  $T_3$  bow space  $(G^*; \mathfrak{B}^*)$  is the completion of  $T_3$  bow space  $(G; \mathfrak{B})$ ,

(G<sub>2</sub>)  $(G; \mathfrak{B})$  is sub-algebraic system of  $(G^*; \mathfrak{B}^*)$ .

**LEMMA.** *Assume that a  $T_2$  space  $(X; \mathfrak{B})$  has its completion  $(X^*; \mathfrak{B}^*)$  which itself is  $T_2$  space. Then  $(X^*; \mathfrak{B}^*)$  is a  $T_3$  space, if and only if for any minimal Cauchy filter  $\dagger$  in  $X$  and for arbitrary  $W \in \dagger$ , there exists  $V \in \dagger$  such that for every Cauchy filter of in  $X$  if  $V \in \mathfrak{g}$  then  $W$  belongs to the minimal Cauchy filter contained in  $\mathfrak{g}$ .*

**THEOREM 9.** *There exists a completion of bow algebraic system  $(G; \mathfrak{B})$  if and only if;*

(E<sub>1</sub>)  $(G; \mathfrak{B})$  satisfies the conditions 1, 2, of author's paper [3],

(E<sub>2</sub>) for every Cauchy filter  $\dagger$ , in  $(G; \mathfrak{B})$  and for every composition  $\pi$  defined in  $(G; \mathfrak{B})$ , the filter  $\dagger\pi\mathfrak{g}$  is a Cauchy filter,

(E<sub>3</sub>)  $(G; \mathfrak{B})$  satisfies the conditions of lemma and proposition,

(E<sub>4</sub>) for any mapping  $f$  defined on it as algebraic system and for any leg  $\dagger$  in  $G$ ,  $\{f(A) \mid A \in \dagger\}$  generates a Cauchy filter in  $G$ .