

98. The Equivalence of Two Methods of Absolute Summability

By A. V. V. IYER

Holkar Science College, Indore, India

(Comm. by Kinjirô KUNUGI, M.J.A., Sept. 12, 1963)

1.1. Definitions. Let $\sum a_n$ be given infinite series with the sequence of partial sums $\{s_n\}$. Let $\{p_n\}$ be a sequence of constants, real or complex, and let us write

$$P_n = p_0 + p_1 + \cdots + p_n.$$

The sequence-to-sequence transformation:

$$t_n = \frac{\sum_{\nu=0}^n p_{n-\nu} s_\nu}{P_n} \quad (P_n \neq 0)$$

defines the sequence $\{t_n\}$ of Nörlund means¹⁾ of the sequence $\{s_n\}$ generated by the sequence of coefficients $\{p_n\}$. The series $\sum a_n$ is said to be summable (N, p_n) to the sum s , if $\lim_{n \rightarrow \infty} t_n$ exists and is equal to s , and is said to be absolutely summable $\{N, p_n\}$, or summable $|N, p_n|$,²⁾ if the sequence $\{t_n\}$ is of bounded variation, that is, the series $\sum |t_n - t_{n-1}|$ is convergent.

In the special case in which $p_n = \frac{1}{n+1}$, we write σ_n for t_n , so that

$$(1.1.1) \quad \sigma_n = \frac{\sum_{\nu=0}^n \frac{s_\nu}{n+1-\nu}}{1 + \frac{1}{2} + \cdots + \frac{1}{n+1}}.$$

We define

$$(1.1.2) \quad \sigma_n^* = \frac{1}{\log n} \sum_{\nu=0}^n \frac{s_\nu}{n+1-\nu}.$$

We shall say that $\sum a_n$ is summable H to s , if $\lim_{n \rightarrow \infty} \sigma_n$ exists and is equal to s , and that $\sum a_n$ is absolutely summable H or summable $|H|$ if $\{\sigma_n\}$ is of bounded variation. Similarly, if $\lim_{n \rightarrow \infty} \sigma_n^* = s$, we shall say that $\sum a_n$ is summable H^* , and if $\{\sigma_n^*\}$ is of bounded variation, we shall say that $\sum a_n$ is absolutely summable H^* or summable $|H^*|$.³⁾

1.2. Introduction. It is easy to establish that for ordinary

1) Nörlund [1]; a definition substantially the same as that of Nörlund was given by G. F. Woronoi in the Proceedings of the 11th Congress of Russian naturalists and scientists (in Russian), St. Petersburg, 60-61 (1902). The first English translation of this work of Woronoi with "remarks of the translator" by J. D. Tamarkin is contained in Woronoi [2].

2) Mears [3].

3) Summability H^* is the same as Harmonic Summability as defined by Riesz [4].